Which way for Wireless Power: High Q or High k?

Bruce Long, John Miller, Andrew Daga, and Peter Schrafel
Momentum Dynamics

Problem Statement

- The present technical paradigm for resonant induction wireless vehicle charging features a small receiving coil on the vehicle matched to a larger ground side sending.
- Multiple advantages attributed to small receiving coils.
- They fit better in the crowded vehicle underbody.
- Less material.
- They weight less, cost less.
- Everybody likes small coils.

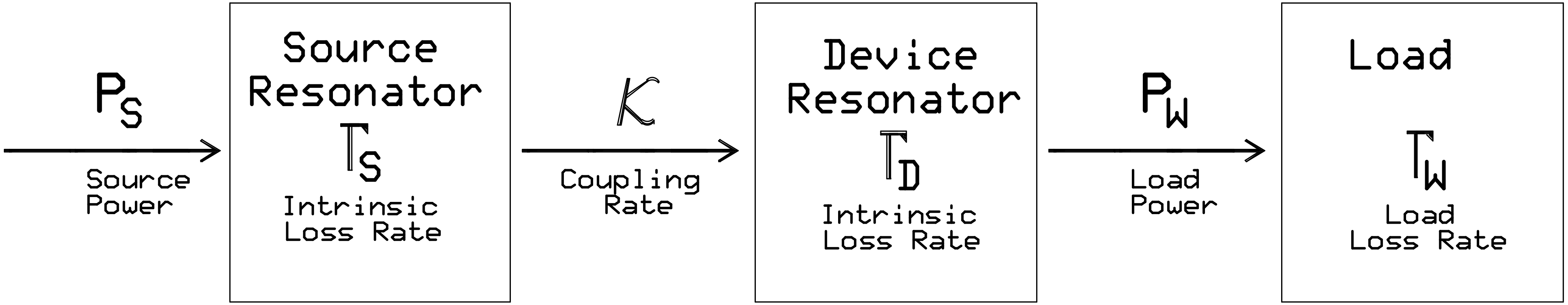
Problem Statement

- But small receiving coils have one significant disadvantage
- Lower coupling coefficient.
- Popular perception is the loss in coupling coefficient can be made up by use of high Q obtained in part by increased operating frequency.

- What does the Q operating frequency, energy transfer efficiency, coil diameter trade space look like?
- What is the engineering trade-off for smaller vehicle side coils?

Coupled Mode Theory

- Because the current vehicle WPT paradigm and reasoning builds from a basis of Coupled Mode Theory (CMT) we will build our discussion on the basis of the equation for power transfer efficient η derived from coupled mode theory.



Coupled Mode Theory

- Coupled resonator theory gives the following expression for the efficiency of resonant induction wireless power transfer:
- $$\eta = \frac{\frac{\Gamma_W}{\Gamma_D}\frac{\kappa^2}{\Gamma_S \Gamma_D}}{\left[\left(1+\frac{\Gamma_W}{\Gamma_D}\right)\frac{\kappa^2}{\Gamma_S \Gamma_D}\right]+\left(1+\frac{\Gamma_W}{\Gamma_D}\right)^2}$$
- Γ_S, Γ_D and Γ_W are the resonance widths or intrinsic rate of decay for the Source, Device and Load resonances.
- κ is the inter-resonant coupling *rate.*
- $\sqrt{\frac{\kappa^2}{\Gamma_S \Gamma_D}}$ is the inter-resonator coupling figure of merit (FOM)

Coupled Mode Theory

- Coupled mode algebra is less intuitive to those having long experience with circuit analysis examination of resonant circuits.
- Fortunately coupled mode expressions can be converted to the more familiar circuit analysis based equations by simple substitution of variables.

Coupled Mode Theory Expression converted to circuit parameters

The CMT resonator coupling rate $\boldsymbol{\kappa}$ becomes: $$\boldsymbol{\kappa} \equiv \frac{\omega M}{2\sqrt{L_S L_P}} = \frac{\omega k}{2}$$

Where $$k = \frac{m}{\sqrt{L_S L_P}}$$

The resonator intrinsic rate of decay becomes: $$\Gamma \equiv \frac{\omega}{2Q} = \frac{\omega}{2\frac{\omega L}{R}} = \frac{R}{2L}$$

For equal primary and secondary inductances: $$\frac{\Gamma_W}{\Gamma_D} = \frac{\frac{R_W}{2L}}{\frac{R_D}{2L}} = \frac{R_W}{R_D}$$

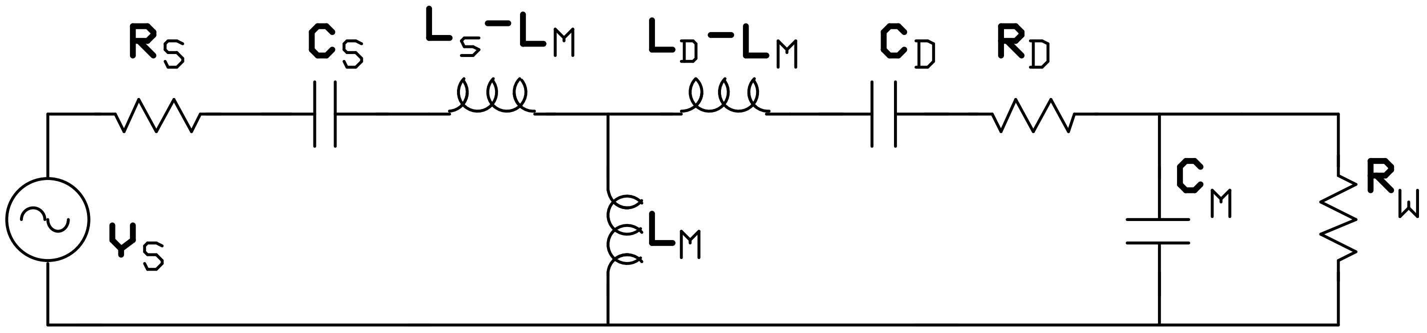
The coupled resonator figure of merit becomes: $$\frac{\kappa^2}{\Gamma_S \Gamma_D} \equiv \frac{\left(\frac{\omega k}{2}\right)^2}{\frac{R_S R_D}{2L\ 2L}} = \frac{(\omega L k)^2}{R_S R_D}$$

Coupled Mode Theory Expression
Converted to circuit parameters

- Transfer efficiency η becomes:

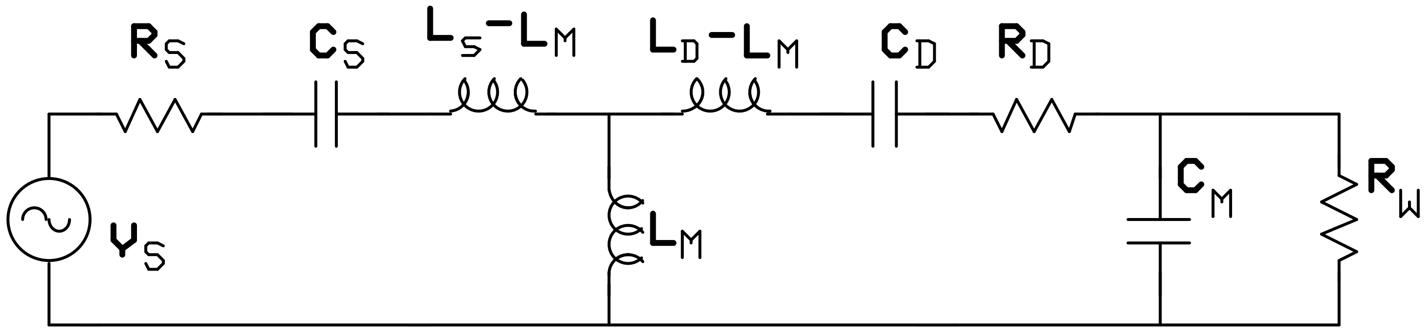
$$\eta = \frac{R_W\,(\omega Lk)^2}{R_S R_D^2 + 2R_S R_D R_W + R_D(\omega Lk)^2 + R_S R_W^2 \ + R_W(\omega Lk)^2}$$

For equivalent circuit:



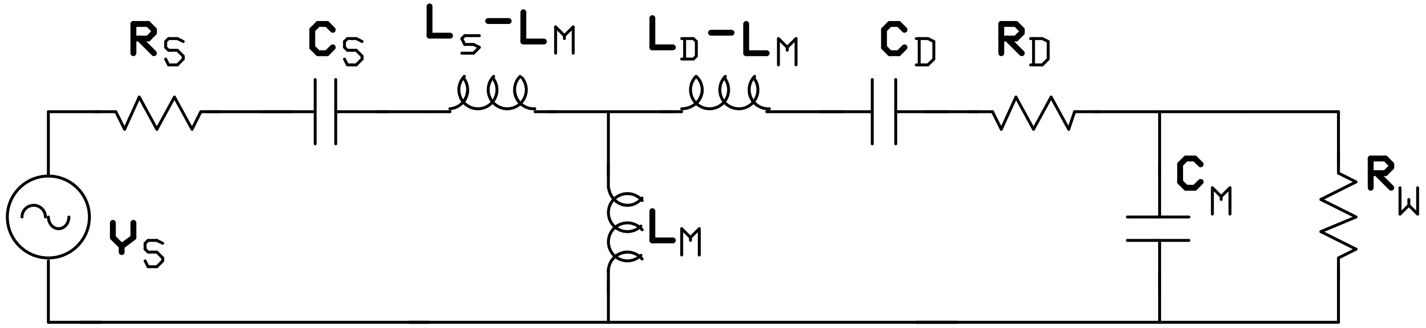
Resonating the coupling coil network

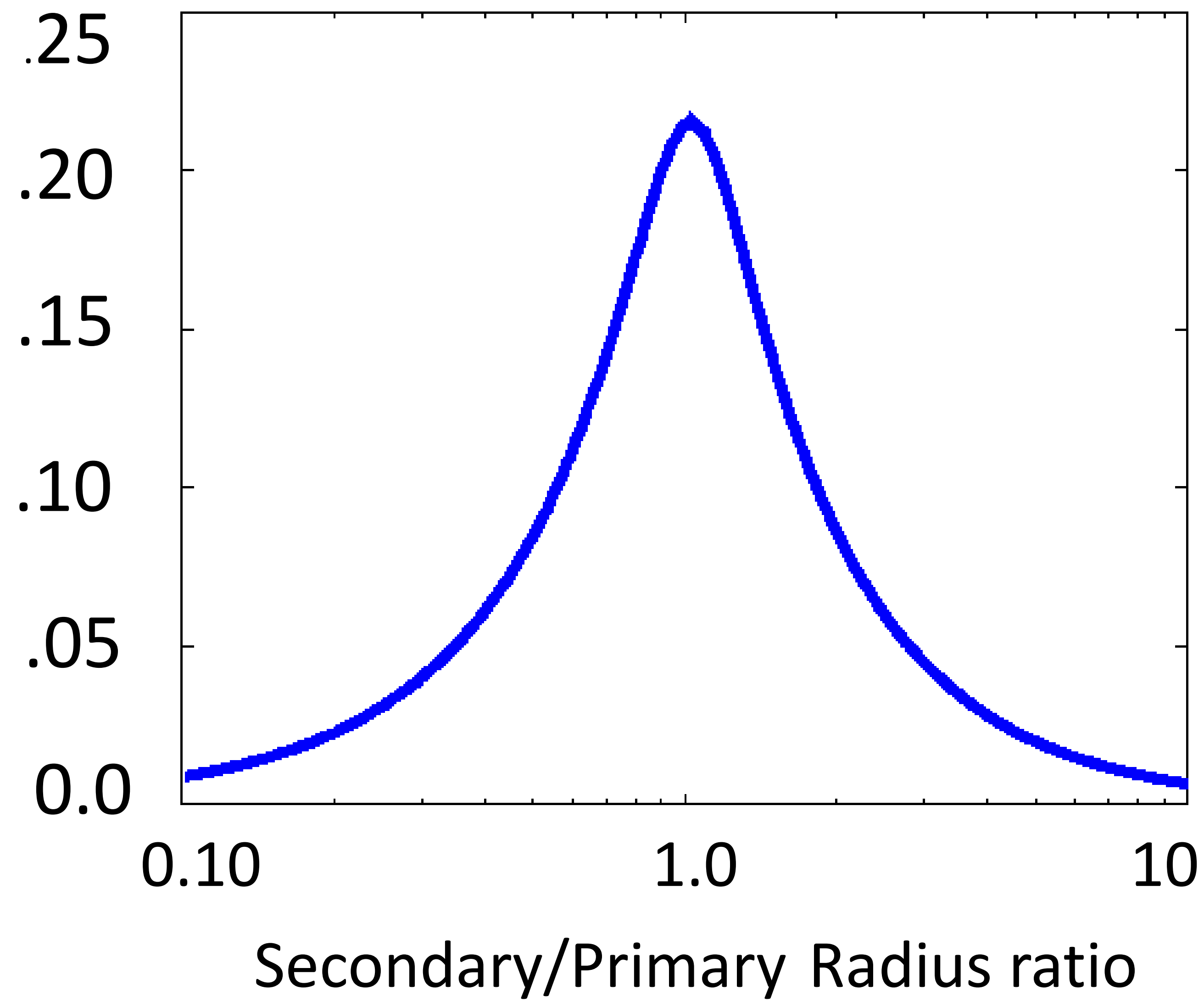
- Series capacitors C_S and C_D resonant with series inductors L_S-L_M and L_D-L_M respectively.
- At resonance the reactance of both series LC branch is zero leaving only the series loss resistors, R_S and R_D in the circuit.
- Shunt resonating capacitor C_M resonates with shunt inductor Lm at resonance creating a parallel LC resonant circuit



Resonating the coupling coil network

- Each inductor resonates with one resonating capacitor
- L_S-L_m in series with C_M, L_D-L_M in series with C_D, L_M in parallel with C_M
- At resonance all reactances are canceled leaving a purely resistive network.
- This resonant network does not provide impedance transformation. The voltage transfer ratio, Vout/Vin is 1:1.
- Circuit analysis can be done by inspection.





Coupling coefficient vs coil radius ratio

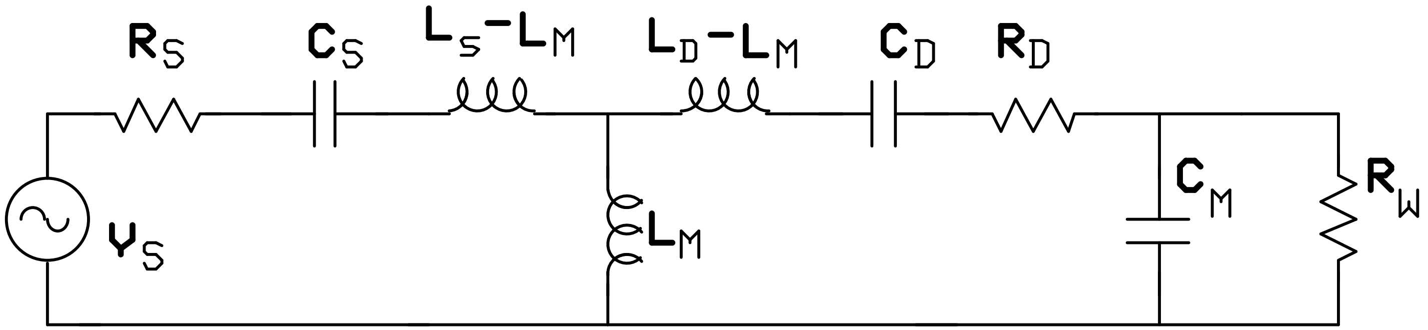
Vehicle Coil Size Reduction
How much does it cost?

- Consider a transfer coil pair consisting of two equal radius spiral coils
- k=.21
- Assume economical design
- Ferrite cross-section just sufficient to avoid saturation
- Conductor cross-section just sufficient to achieve required efficiency

Apply Figure of Merit (FOM)

Figure of Merit = $\frac{(\omega L k)^2}{R_S R_D} = \frac{(Shunt\ Branch\ Impedance)^2}{\sqrt{R_S R_D}^2}$

At resonance $I_{LM} = V_{RW} / (\omega L_M k)$ V_{RW} fixed by $V_{BATTERY}$



Vehicle Coil Radius Reduction
How much do you lose?

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- Now reduce the vehicle coil radius by 30%
- r' = .70 r and from Figure   k' = .15   k' =.7k = .7(.21)
- **Radius reduction reduces coupling coefficient by factor of .7**
- In a flat spiral coil $L \propto (\text{Area})N^2$
- Radius reduction reduces area by factor of $.7^2 = .5$
- Radius reduction reduces number of turns by factor of .7
- $L' = (.7)^2\,(.7)^2\,L$ $L' = .25L$
- **Radius reduction reduces vehicle coil inductance by factor of 4**

Vehicle Coil Radius Reduction
How much FoM do you lose?

- For LS ≠ LD $$FoM = \frac{(\omega k)^2 L_S L_D}{R_S R_D}$$

- $$FOM' = \frac{(\omega(.7)k)^2 L_S (.25) L_D}{R_S (.7) R_D} = .175\ FoM$$

- Credit given for reduced length of vehicle coil conductor
- **Reducing vehicle side coil radius by 30% reduces FoM by factor 5.7**

Vehicle Coil Radius Reduction
Effect upon L_M branch impedance?

- $L_m = k\sqrt{L_S L_D}$ $L_m = (.7)k\sqrt{L_S(.25)L_D}$ $L'_m = (.7)(.5)L_M = .35L_M$

- Impedance of L_M branch reduced by = .35
- Shunt branch current must increase by 1/.35 = 2.86 to maintain Vbat
- **Conductor cross-section must be increased by factor of 2.86 to maintain efficiency.**
- **Ferrite cross-section must be increased by factor of 2.86 to avoid saturation with the increased current.**
- **Will conductor and ferrite fit in reduced sized enclosure?**
- **Weight and cost reduction doubtful.**

Conclusions

- **Vehicle coil radius reduction is very, painful.**
- **Weight and cost benefits are unlikely.**
- **Weight and cost go down if vehicle coil radius is increased.**