Which way for Wireless Power: High Q or High k?

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Problem Statement

• The present technical paradigm for resonant induction wireless vehicle charging features a small receiving coil on the vehicle matched to a larger ground side sending.

• Multiple advantages attributed to small receiving coils.
  • They fit better in the crowded vehicle underbody.
  • Less material.
  • They weight less, cost less.
  • Everybody likes small coils.

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• But small receiving coils have one significant disadvantage
• Lower coupling coefficient.
• Popular perception is the loss in coupling coefficient can be made up by use of high Q obtained in part by increased operating frequency.

• What does the Q operating frequency, energy transfer efficiency, coil diameter trade space look like?
• What is the engineering trade-off for smaller vehicle side coils?
Because the current vehicle WPT paradigm and reasoning builds from a basis of Coupled Mode Theory (CMT) we will build our discussion on the basis of the equation for power transfer efficient $\eta$ derived from coupled mode theory.
Coupled Mode Theory

• Coupled resonator theory gives the following expression for the efficiency of resonant induction wireless power transfer:

\[ \eta = \frac{\frac{\Gamma_W \kappa^2}{\Gamma_D \Gamma_S \Gamma_D}}{\left[\left(1 + \frac{\Gamma_W}{\Gamma_D}\right) \frac{\kappa^2}{\Gamma_S \Gamma_D}\right] + \left(1 + \frac{\Gamma_W}{\Gamma_D}\right)^2} \]

• \( \Gamma_S, \Gamma_D \) and \( \Gamma_W \) are the resonance widths or intrinsic rate of decay for the Source, Device and Load resonances.

• \( \kappa \) is the inter-resonant coupling rate.

• \( \sqrt{\frac{\kappa^2}{\Gamma_S \Gamma_D}} \) is the inter-resonator coupling figure of merit (FOM)
Coupled Mode Theory

- Coupled mode algebra is less intuitive to those having long experience with circuit analysis examination of resonant circuits.
- Fortunately coupled mode expressions can be converted to the more familiar circuit analysis based equations by simple substitution of variables.
Coupled Mode Theory Expression converted to circuit parameters

The CMT resonator coupling rate $\kappa$ becomes:

$$\kappa \equiv \frac{\omega M}{2\sqrt{L_S L_P}} = \frac{\omega k}{2}$$

Where

$$k = \frac{m}{\sqrt{L_S L_P}}$$

The resonator intrinsic rate of decay becomes:

$$\Gamma \equiv \frac{\omega}{2Q} = \frac{\omega}{2\omega L} = \frac{R}{2L}$$

For equal primary and secondary inductances:

$$\frac{\Gamma_W}{\Gamma_D} = \frac{\frac{R_W}{2L}}{\frac{R_D}{2L}} = \frac{R_W}{R_D}$$

The coupled resonator figure of merit becomes:

$$\frac{\kappa^2}{\Gamma_S \Gamma_D} \equiv \frac{\left(\frac{\omega k}{2}\right)^2}{\frac{R_S R_D}{2L 2L}} = \frac{(\omega Lk)^2}{R_S R_D}$$
Coupled Mode Theory Expression
Converted to circuit parameters

- Transfer efficiency $\eta$ becomes:

$$\eta = \frac{R_W (\omega L k)^2}{R_S R_D^2 + 2R_S R_D R_W + R_D (\omega L k)^2 + R_S R_W^2 + R_W (\omega L k)^2}$$

For equivalent circuit:
Resonating the coupling coil network

- At resonance the reactance of both series LC branch is zero leaving only the series loss resistors, $R_S$ and $R_D$ in the circuit.
- Shunt resonating capacitor $C_M$ resonates with shunt inductor $L_m$ at resonance creating a parallel LC resonant circuit
Resonating the coupling coil network

- Each inductor resonates with one resonating capacitor
- \( L_s - L_m \) in series with \( C_M \), \( L_D - L_M \) in series with \( C_D \), \( L_M \) in parallel with \( C_M \)
- At resonance all reactances are canceled leaving a purely resistive network.
- This resonant network does not provide impedance transformation. The voltage transfer ratio, \( V_{out}/V_{in} \) is 1:1.
- Circuit analysis can be done by inspection.
Coupling coefficient vs coil radius ratio

Secondary/Primary Radius ratio

Coupling coefficient vs coil radius ratio
Vehicle Coil Size Reduction
How much does it cost?

- Consider a transfer coil pair consisting of two equal radius spiral coils
- $k = 0.21$
- Assume economical design
- Ferrite cross-section just sufficient to avoid saturation
- Conductor cross-section just sufficient to achieve required efficiency
Apply Figure of Merit (FOM)

Figure of Merit = \( \frac{(\omega Lk)^2}{R_S R_D} = \frac{(\text{Shunt Branch Impedance})^2}{\sqrt{R_S R_D}^2} \)

At resonance \( I_{LM} = \frac{V_{RW}}{(\omega L_M k)} \) \( V_{RW} \) fixed by \( V_{\text{BATTERY}} \)
Vehicle Coil Radius Reduction
How much do you lose?

- Now reduce the vehicle coil radius by 30%
- \( r' = 0.70 \, r \) and from Figure \( k' = 0.15 \)  \( k' = 0.7k = 0.7(0.21) \)
- **Radius reduction reduces coupling coefficient by factor of 0.7**
- In a flat spiral coil \( L \propto (\text{Area})N^2 \)
- Radius reduction reduces area by factor of \( 0.7^2 = 0.5 \)
- Radius reduction reduces number of turns by factor of 0.7
- \( L' = (0.7)^2 \times (0.7)^2 \, L \quad L' = 0.25L \)
- **Radius reduction reduces vehicle coil inductance by factor of 4**
Vehicle Coil Radius Reduction
How much FoM do you lose?

- For $LS \neq LD$
  \[ F_{oM} = \frac{(\omega k)^2 L_S L_D}{R_S R_D} \]

- $F_{oM}' = \frac{(\omega(.7)k)^2 L_S(.25)L_D}{R_S(.7)R_D} = .175 F_{oM}$

- Credit given for reduced length of vehicle coil conductor

- **Reducing vehicle side coil radius by 30% reduces FoM by factor 5.7**
Vehicle Coil Radius Reduction
Effect upon $L_M$ branch impedance?

- $L_m = k \sqrt{L_S L_D}$
- $L_m = (0.7)k \sqrt{L_S (0.25)L_D}$
- $L'_m = (0.7)(0.5)L_M = 0.35L_M$

- Impedance of $L_M$ branch reduced by $= 0.35$
- Shunt branch current must increase by $1/0.35 = 2.86$ to maintain $V_{bat}$
- Conductor cross-section must be increased by factor of 2.86 to maintain efficiency.
- Ferrite cross-section must be increased by factor of 2.86 to avoid saturation with the increased current.
- Will conductor and ferrite fit in reduced sized enclosure?
- Weight and cost reduction doubtful.
Conclusions

• Vehicle coil radius reduction is very, painful.

• Weight and cost benefits are unlikely.

• Weight and cost go down if vehicle coil radius is increased.