Core Loss Modeling

Edward Herbert
Biography: Edward Herbert

Ed earned a Bachelor of Engineering degree in Electrical Engineering from Yale University, Class of 1963. He worked as a design engineer, a project engineer, an engineering supervisor, then as engineering manager until 1985. Since then, Ed had been independent, promoting patented technology for license.

Within PSMA, Ed is a member of the PSMA Board of Directors, Co-Chairman of the Magnetics Committee and Co-Chairman of the Energy Efficiency Committee.

Ed was a champion of the core loss studies at Dartmouth.
Core Loss Parameters

To characterize a core material, a core or a wound component, more parameters are needed. Further, they need to be characterized at different operating conditions, particularly temperature.

An anecdote:

We were helping to design a power transformer for 2 kW, 2 MHz square-wave operation. When tested, the transformer experienced thermal run-away, suggesting that some lossy parameter had a positive temperature coefficient.

Nothing in the data sheets showed that there might be a problem.

The lossy parameters may include the resistivity, $\rho$, and the imaginary part of the permeability, $\mu''$, and the imaginary part of the permittivity, $\varepsilon''$. 

Hysteresis losses are ignored for now, as they tend to be negligible at very high frequency.
Determining core loss parameters

The core loss material parameters $\rho$, $\mu'$, $\mu''$, $\varepsilon'$ and $\varepsilon''$ are not measured directly.

For example, to determine resistivity, $\rho$, first the resistance $R$ of a sample is found, then dimensions are applied to convert $R$ to $\rho$, a bulk material property.

For many uses, this conversion is not necessary. To characterize a specific core or wound component, often it is more useful to specify the resistance, $R$.

The permeability, $\mu$, and the permittivity, $\varepsilon$, are first measured as inductance, $L$, and capacitance, $C$.

The inductance and capacitance of ferrite are very non-ideal, so their lossy components need to be found as resistances. The respective values of the resistors, inductors and capacitors allows the permeability, $\mu$, and the permittivity, $\varepsilon$, to be resolved into their real and imaginary parts, $\mu'$, $\mu''$, $\varepsilon'$ and $\varepsilon''$.

In this discussion, we will not attempt to apply the dimensions. We will use $R$, $L$ and $C$. 
Avoid frequency, $f$, and maximum flux density, $\hat{B}$.

A secondary objective of this effort is to find a SPICE model for core loss. Neither frequency, $f$, nor maximum flux density, $\hat{B}$ can be used in SPICE, because neither can be determined as an instantaneous parameter as a simulation is running.

Avoid flux, $\Phi$, and flux density, $B$.

Flux, $\Phi$, is a mixed parameter, the product of volts, $V$, and time, $s$. Use volts, $V$, and time, $s$, separately. We need to vary them independently.

The flux density, $B$, is even worse, because it mixes in dimensions.
Steinmetz equation:

\[ P = k \cdot f^a \cdot \hat{B}^b \]

Problems:
- An objective is a SPICE model. Neither \( f \) nor \( \hat{B} \) can be used in SPICE.
- The independent variables \( V \) and \( t \) are intermingled in \( \hat{B} \). We need to vary them independently.

For square-wave excitation

\[ f = \frac{1}{2 \cdot t} \quad \hat{B} = \frac{V \cdot t}{A} \]

The Steinmetz equation can be rewritten as

\[ \frac{W}{Vol} = k \cdot (2 \cdot t)^{-a} \cdot \left(\frac{V \cdot t}{A}\right)^b \]

Substituting and rearranging:

\[ W = k' \cdot t^{b-a} \cdot V^b \]

\( P \) is core loss, W/m\(^3\)
\( k \) is a constant (NOT !)
\( f \) is frequency
\( \hat{B} \) is the maximum flux density
\( a \) is a constant exponent (NOT !)
\( b \) is a constant exponent (NOT !)

\( t \) is pulse width, s
2*\( t \) is the period (square-wave)
\( V \) is voltage per turn
\( A \) is the core area, m\(^2\)
\( Vol \) is the core volume, m\(^3\)

[Put the core geometry into \( k' \) so the answer reads directly as Watts]

\[ k' = k \cdot 2^{-a} \cdot A^{-b} \cdot Vol \]

The independent variables \( V, t \), are now separated.
The Herbert graph

This example of the Herbert graph shows the core loss for three wound components with 5 turns. (Red, Green, Blue) The x-axis is the pulse width of the square-wave \[ f = 1/(2\pi t) \] The y-axis is core loss in watts, W. The curves are the square-wave voltage, V, applied at the terminals.

A graph for a specific core should use volts per turn, V/n
A graph for a material should use V/(n*m^2) and the result is in W/m^3
These are scale factors, and do not change the shape of the curves.

[The dashed red lines are constant volt-seconds, v-s. They can be scaled to \( \dot{B} \) by dividing by n*A_e.]

[The odd lines descending to the right show the core loss at low duty-ratio. That is beyond the scope of this presentation.]
The “Composite waveform hypothesis”

The original impetus for the Pilot Project Core Loss study at Dartmouth was to test and analyze the composite waveform hypothesis. The conclusion by Dr. Sullivan was:

“Despite the minor discrepancies, the loss prediction method yields higher accuracy, and is easier to use, than other methods for non-sinusoidal waveforms.”

To use the Herbert curve to determine the core loss for a square wave, find the pulse width on the X-axis, draw a vertical line to the curve that is the square wave excitation voltage, then draw a horizontal line to the Y-axis. Read the core loss directly in Watts.

The Herbert curve can be used to find the approximate loss for low duty-ratio rectangular wave excitation. A recent addition adds a provision for off-time. These are not the topic of this presentation. More information can be found on the PSMA web site, Composite Waveform Hypothesis.
Admittance curves

Divide the curves in the Herbert graph by $V^2$.

The idea is that $G = p/V^2$, so we are exploring an admittance function. $G = 1/Z$.

The result is as shown on the right.

The spacing along the Magenta line is fairly even.

If the admittance curves are moved along the Magenta line, it can be seen that they all have the same shape and nest well.

This gives some optimism that a single admittance function $G$ can define the curves.
Core loss approximation

\[ W = k \cdot t^d \cdot \left(1 + \left(\frac{V}{V_b}\right)^a \cdot \left(\frac{t}{t_b}\right)^b\right) \cdot V^2 \]

Where

- \( W \) is the core loss in Watts
- \( t \) is the square-wave pulse width in us
- \( k, a, b, \) and \( d \) are constants (not the Steinmetz constants)
- \( V_b \) is the base line voltage, 1.25 in this example
- \( t_b \) is the break-point time in us, 5.2 in this example

- \( k = 0.002 \)
- \( d = -0.65 \), the slope at the left
- \( a = 1 \)
- \( b = 2.5 \) It is the slope 1.85 minus the slope -0.65

The red lines and points are data

The green lines are the approximation
Using SPICE to extract parameters

If one equation works over the entire range, maybe a SPICE model can work over the entire range.

The easiest to implement would have the form

\[ G = G_1 + G_2 + \cdots + G_n. \]

Networks of resistors, capacitors and inductors will approximate the losses and reactances using actual test data as the voltage source.

The data are from the PSMA-Dartmouth core loss studies

We will start with the hysteresis loop for test run mi12-2-01, \( V = 1.25 \) V and \( t = 1.0 \) us.
Hysteresis loop, mi012-2-001

With square wave excitation, the y-axis can be scaled as time.
\[ B = \frac{V \cdot t}{n \cdot A} \] (V, n and A are constants.)

The x-axis can be scaled as current, I.
\[ H = \frac{n \cdot I}{\ell} \] (\(\ell\) and n are constants.)

The hysteresis loop is ragged because the input voltage is ringing.

\[ V = 1.25 \text{ V square-wave} \]
\[ t = 1.0 \text{ us} \]
\[ (f = 500 \text{kHz}) \]
Hysteresis loops of common components:

- **Inductor**: Straight line passing through 0, 0 width, no loss
- **Resistor**: Straight sides, symmetrical about 0 Width, Losses
- **Resistor-capacitor**: $RC \approx t$, sides curved in Width, Losses
- **Resistor-capacitor**: $RC << t$, peaks at top and bottom, like switching losses
Adding currents to synthesize a hysteresis loop
Subtracting currents to model a hysteresis loop

Test data

Simulated Inductor current

Simulation Error

\[ I = I_{\text{her}} - I_{\text{hx}} \]
Improving the SPICE model

Using the same excitation $V$, vary the components in the SPICE model until the current $I_{hx}$ equals the test data for the core. This gives you the core parameters for very high frequencies.
Four points

Blue curves are data. Red are SPICE.
Four points
These are the same four points drawn to the same scale.

Note that mi2-2-037 has the highest v-s.

-037 also shows more of the low frequency influence (hysteresis loss). The model is too skinny because we have not yet added the hysteresis loss algorithm.

Blue curves are data. Red are SPICE.
More points
The low frequency influence is seen
For very high frequency use, the network shown using Resistors, Capacitors and an Inductor is suitable as a simple SPICE model. Their values can be used to determine $\rho$, $\mu'$, $\mu''$, $\varepsilon'$ and $\varepsilon''$.

The low frequency asymptote (hysteresis loss) are not nearly as straight-forward.

Questions?