Transient Analysis of Large-Can Aluminum Electrolytic Capacitors

Sam G. Parler, Jr., P.E.
Director of Research and Development
Outline:

I. Consequences and analysis techniques of transient behavior peculiar to large aluminum electrolytic capacitors:
   1. Time-varying hotspot location
   2. Highly anisotropic winding
   3. Coaxial microstructure and non-uniformly distributed RC behavior

II. Calculating capacitor lifetime with time-varying stressors:
   1. Why arithmetic averaging doesn’t work
   2. Development of nonlinear averaging technique
   3. Example of application to discrete stressor distribution
   4. Example of application to continuous stressor distribution
Distinguishing feature #1: Time-varying hotspot location

\[
\frac{\text{TabMass}}{\text{TabR}} < \frac{\text{Winding Mass}}{\text{Winding R}}
\]

Spatial distribution of transient response to current pulse will be much different from that of the steady state response.

Steady state response to 30 amps rms in 45 °C ambient shows tabs running no hotter than winding:
Time-varying hotspot location

Transient Response to 16 kA pulse at 45 ºC initial temperature and subsequent cool-down shows <0.1 ºC heat rise in winding but 5 ºC heat rise in the aluminum tabs:

The tab cool-down curve from initial 10 ºC step shows a time constant of 7 seconds.
Time-varying hotspot location

Modeling approaches include FEA as previously shown, or lumped-element Spice model as shown in the thermal circuit below.

Thermal $\rightarrow$ Electrical
$mC_P \rightarrow C$
$\theta \rightarrow R$
$P \rightarrow I$
$T \rightarrow V$
Distinguishing feature #2: Highly anisotropic winding

The winding is of cellulosic separators and aluminum foils, and thus exhibits thermal anisotropicity over two orders of magnitude higher conductivity in the axial than in the radial direction. This behavior can be captured in FEA using diagonal tensor conductivity matrix, e.g. $\{0.6,0.6,100\}$, or approximated in Spice by using separate axial and radial thermal resistances. Note: Fixed radial conductivity valid only for large capacitors.

Nota bene: The can wall is not isothermal!
Highly anisotropic winding

Below is a comparison of a metallized polypropylene film capacitor and an aluminum electrolytic capacitor, both approx 76x120 mm and dissipating 5 watts in 45 °C environment.
Distinguishing feature #3: Coaxial microstructure and distributed RC behavior

The electrolyte is an ionically conductive fluid; this is not the dielectric. The dielectric is aluminum oxide $\text{Al}_2\text{O}_3$ which is grown in an anodizing bath upon the tortuous surface of a highly etched aluminum foil.

In an aluminum electrolytic capacitor, the (+) conductor is the aluminum anode foil and the (-) is the electrolyte.

Note: In the micrographs below, the aluminum surrounding the alumina dielectric tubes has been dissolved away.
Coaxial microstructure and distributed RC behavior

**URC Analytic or Spice Element as a first approximation:**

\[
Z_{in} = r \sqrt{\frac{r}{j \omega c}} \coth \sqrt{j \omega c L^2}
\]

Units:
- \( r \) [Ω/m]
- \( c \) [F/m]
- \( L \) [m]

Shortcoming:
Above model predicts \( R \) & \( C \) to fall as \( 1/\sqrt{f} \) (Warburg element) while typical behavior is usually closer to \( 1/f \).
Transient modeling of non-uniformly distributed RC behavior

Potential improvements to uniform RC line model: Finite-length tapered RC line

PSpice implementation?
- Non-uniform / tapered lines not directly supported in PSpice
- While exact expressions in s domain exist for some taper profiles, they generally involve Bessel and Kelvin functions, which make it impossible to implement directly as ABM “ELAPLACE” component.
- Possible approximation: Use PSpice’s EFREQ source with (freq, mag, phase) list as shown on next slide.

Other Approaches:
- FEA easily accommodates arbitrary taper profiles
- FEA shape-optimization tools can find a best-fit taper
Transient modeling of non-uniformly distributed RC behavior

- Data taken at -40 °C in chamber with Z bridge controlled with VB.net
- Data stored in spreadsheet automatically using VB.net (see below)
- Spreadsheet data converted to Spice-compatible \((f, |Z|, \theta)\) listing using spreadsheet text functions
- Cut and paste into PSpice .cir file seen at right
- PSpice graphical outputs seen at lower right
- Data and simulate match well
- Next step is to apply to transient excitation

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Spice Test Analysis

```plaintext
Vp 1 0 AC 1V
Rs 1 2 1
E1 2 0 FREQ {(V(1)-V(2)} WAG RAD
+ 10.69 729 067 02153 1 1242 592 942 3611
+ 20.39 477 397 12359 1 020 500 512 45
+ 40.26 245 115 543 935 1 073 376 420 936
+ 100.20 229 087 071 0672 1 402 464 759 736
+ 120.19 656 305 571 785 1 359 944 653 1619
+ 200.19 641 517 541 433 1 262 005 646 1101
+ 400.17 754 522 030 1004 1 207 423 360 4906
+ 1000.16 619 233 020 335 1 227 020 379 0176
+ 2000.15 151 420 508 968 1 293 431 292 6208
+ 4000.13 759 541 487 55 1 394 926 726 1311
+ 10000.10 202 119 813 4604 1 696 968 779 4769
+ 20000.7 779 152 252 615 1 696 026 070 6854
+ 40000.5 6 597 066 227 0599 1 840 970 326 9562
+ 100000.3 292 659 300 6564 1 055 289 219 7278
+ 200000.2 052 342 469 3127 1 240 023 450 5123
+ 400000.1 218 274 894 429 1 401 400 954 2717
+ 500000.0 015 492 310 722 1 567 284 729 9999
```

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Data and Representative Model Points: \(R, C, \theta, |Z|\) vs. Frequency & Temperature
Transient modeling of non-uniformly distributed RC behavior

There is a problem in Spice:
* Warning messages of non-causality
* Message advices to delay excitation by at least 34.5 ms

However:
* Delaying excitation by 100 ms does not fix the problem
* Warning message still occurs
* Response waveform is not correct
Alternative Approach using FEA Shape-Optimization Techniques:

* Start with rectangular, electrolyte-filled pore with capacitive walls
* Create a “deformed mesh” that allows the vertical pore wall to move radially by \( dr(z) \) where

\[
dr = \sum_{k=1}^{N} q_k d_k \sin k \pi s
\]

* Minimize an objective function representing the impedance error, e.g.:

\[
\left( \frac{R}{R_{\text{TARGET}}} - \frac{R_{\text{TARGET}}}{R} \right)^2 + \left( \frac{C}{C_{\text{TARGET}}} - \frac{C_{\text{TARGET}}}{C} \right)^2
\]
Transient modeling of non-uniformly distributed RC behavior

Physical Model Assures Causality

Voltage varying with zero terminal current is typical of capacitor exhibiting dielectric relaxation, dielectric loss and "soakage"
Other transient modeling considerations: Calculating lifetime with time-varying stressors

Typical capacitor lifetime equation is:

\[ L = L_B M_V 2^{(T_B - T_C)/10} \]

where

- \( L_B \) = Base lifetime with rated DC voltage applied at \( T_B \) ambient
- \( T_C \) = Core temperature
- \( M_V \) = Voltage multiplier such as \( (V_R/V_A)^3 \) where
  - \( V_R \) = rated DC voltage and
  - \( V_A \) = applied DC voltage (\( V_A \leq V_R \))

Obviously lifetime varies with the applied stressors in a highly nonlinear manner. Thus when the stressors vary with time, arithmetic averaging is not valid. Actual capacitor lifetime will in general be lower than the arithmetic average.

For example, if the capacitor is operating at 25 °C for 12 hours per day and at 85 °C for the other 12 hours per day, the lifetime will be considerably less than the lifetime calculated at a constant 55 °C.

So... How do we calculate the lifetime when the stressors vary with time?
Calculating lifetime with time-varying stressors

\[ L = L_B M_V 2^{(T_B - T_C)/10} \]

We calculate the lifetime when the stressors vary with time as follows:

Let us consider a time varying core temperature \( T_C(t) \).

If \( L \) is the lifetime, then the rate at which the lifetime is consumed is \( 1/L \).

Therefore for each elapsed time element \( dt \), the portion of the total lifetime that is consumed is \( dt/L \). This can be done on an hour-by-hour basis in a spreadsheet, or it can be calculated analytically as follows:

If the stressor \( T_C(t) \) is applied for a representative period \( \tau \) then the lifetime is:

\[
L = \frac{\tau}{\int_0^\tau \frac{dt}{L[stressors(t)]}} = \frac{\tau}{\int_0^\tau \frac{dt}{L[T_C(t)]}}
\]
Calculating lifetime with time-varying stressors

\[ L = L_B M_V 2^{(T_B - T_C)/10} \]

Assume the above has \( L_B = 5 \) kh, \( T_B = 105 \) °C and \( M_V = 1.5 \), then at \( T_C = 55 \) °C the lifetime is

\[ L = 5 \text{ kh} \times 1.5 \times 2^{50/10} = 240 \text{ kh}. \]

Revisiting the problem of a capacitor operating at 25 °C for 12 hours per day and at 85 °C for the other 12 hours per day, we have \( \tau = 24 \) h and

\[ T_C(t) = \begin{cases} 
 25 & 0 \leq t < 12 \\
 85 & 12 \leq t < 24 
\end{cases} \]

\[ L = \frac{\tau}{\int_0^{\tau} \frac{dt}{L[\text{stressors}(t)]}} = \frac{\tau}{\int_0^{\tau} \frac{dt}{L[T_C(t)]}} = \frac{24h}{\int_0^{12} \frac{dt}{7.5kh \times 2^{105-25/10}} + \int_{12}^{24} \frac{dt}{7.5kh \times 2^{105-85/10}}} \]

\[ = \frac{24h}{12h + 12h} = \frac{24h}{1920kh + 30kh} = 59,077 \text{ h} \ll 240 \text{ kh} \]
Example: Calculating lifetime with daily and seasonally varying stressors

\[ L = L_B M_V 2^{(T_B - T_C)/10} \]

Again assume the above has \( L_B = 5 \) kh, \( T_B = 105 \) °C and \( M_V = 1.5 \). Assume average temperature is 65 °C and that the daily temperature and annual variations are sinusoidal, each with 20 °C peak-to-peak.

In *Mathematica* we calculate the lifetime as 95 kh:

\[
\begin{align*}
\text{In}[4]:= & \quad T[t, A, a, b, Tavg] := A \sin(2 \pi a t) + B \sin(2 \pi b t) + Tavg \\
\text{In}[32]:= & \quad L[Tc, Tb, Mv, Lb] := Lb Mv 2^{((Tb - Tc)/10)} \\
\text{In}[45]:= & \quad \text{Life}[A, a, b, Tavg, Tb, Mv, Lb] := \\
& \quad (1. / b) / \text{NIntegrate}[1. / L[T[t, A, a, b, Tavg], Tb, Mv, Lb], \\
& \quad \{t, 0., 1. / b\}] \\
\text{In}[46]:= & \quad \text{Life}[10., 1. / 24., 10., 1. / 8760., 65., 105., 1.5, 5000.] // \text{Quiet} \\
\text{Out}[46]:= & \quad 95025.1
\end{align*}
\]
Summary

There are some properties of large aluminum electrolytic capacitors that differ from small ones, such as:
- Constant radial thermal conductivity
- Relative electrothermal tab stress during large pulse currents

Quite a few properties of aluminum electrolytic capacitors differ from those of metallized film and most other capacitors, including:
- Lossy aluminum tabs
- High thermal anisotropicity
- Large impedance variation with temperature due to electrolyte viscosity
- Large dielectric loss and attendant memory effects
- Distributed RC impedance behavior

The above characteristics can make transient modeling a challenge.

Lifetime models of large aluminum electrolytic capacitors are similar to those of other technologies, but special techniques need to be used in order to model the lifetime under time-varying stressors.
Questions?