

# “Steinmetz-like” Equation for Ferrites

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## ***Approximation formula***

An objective of the Phase II studies sponsored by PSMA at Dartmouth was to find a “Steinmetz-like” equation for core losses in magnetic cores using square wave excitation and “electrical terms,” volts and time. The following formula provides a reasonably accurate approximation of the core losses of one core:

$$P_c = k * T^\delta * \left(1 + \left(\frac{V}{V_b}\right)^\alpha * \left(\frac{T}{T_b}\right)^\beta\right) * V^2$$

Where

- $P_c$  is the core loss, in Watts
- The excitation is a square wave of pulse width  $T$   $\mu$ s and amplitude  $V$  volts.
- The parameters  $k$ ,  $\alpha$ ,  $\beta$  and  $\delta$  are constants (but not the Steinmetz constants).
- $V_b$  is the baseline voltage
- $T_b$  is the beak- point time in seconds.

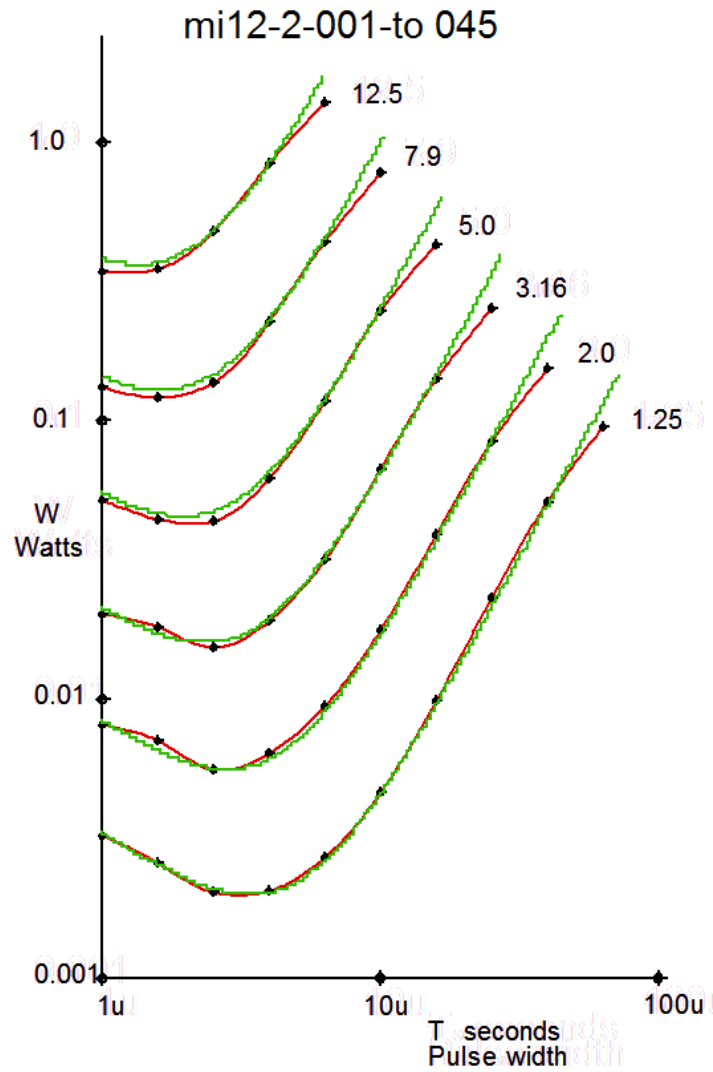
The graph below is the Herbert curves for core loss for a Magnetics Inc. F-42206-TC core with five turns. Because the data were taken for the specific core and winding as a component, the core loss is read directly in Watts on the vertical axis. The horizontal axis is the pulse width  $T$  of the square wave and the lines are the test excitation voltages  $V$  applied at the terminal. (The curves can be expressed as  $V$ /turn by dividing  $V$  by 5.)

The points are data taken during the Phase III core loss study, sponsored by PSMA at Dartmouth, specifically test runs mi12-2-001 through mi12-2-045. The red lines are curves connecting the data points, and the green lines are the approximation given by solving the equation above.

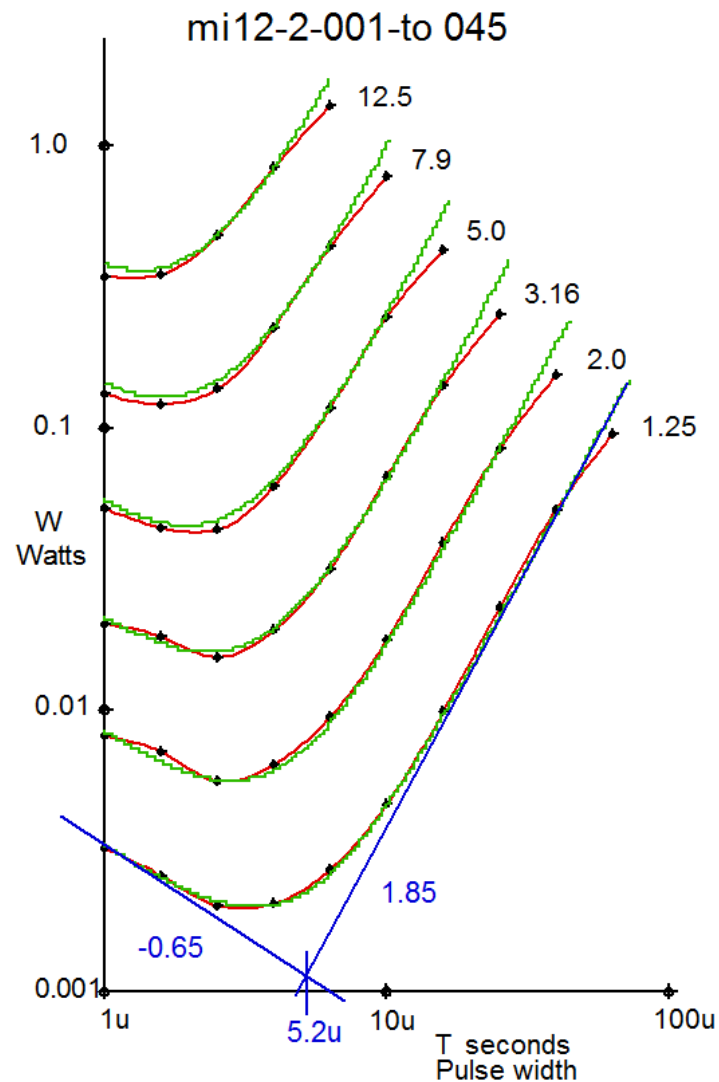
Because  $T$  is defined as the pulse-width, 1  $\mu$ s represents a 500 kHz square wave. The curve fits are quite good until just before saturation (the upper right extreme of each curve). It is likely that a factor could be added to account for impending saturation if it were needed, as it is consistent, curve to curve.

For the F-42206-TC core, the constants are”

- $k = 0.002$
- $\delta = -0.65$
- $\alpha = 1$
- $\beta = 2.5$
- $V_b = 1.25$ , the voltage of the lowest curve
- $T_b = 5.2 \mu\text{s}$ , the break point (intersection of the asymptotes) of the lowest curve.



## Derivation of the parameters



The constant  $\delta = -0.65$  is the slope of the asymptote of the curve on the left in the example above.

The constant  $\beta = 2.5$  is derived from the slope of the asymptote of the curve on the right by subtracting  $\delta = -0.65$ . That is,  $1.85 - (-0.65) = 2.5$

The constant  $T_b$  is the intersect of the asymptotes,  $5.2 \mu$ s.

The constant  $V_b$  is the voltage of the lowest curve, 1.25 V.

The constant  $k$  is calculated by solving the equation for a known point. From the data from the test run mi12-2-001, the core loss is 0.00324 W. We do not yet know the

constant  $\alpha$ , but since we are working with the lowest curve,  $V = 1.25$ , and  $V/V_b = 1$ , so the exponent is unimportant. Let  $\alpha = 1$  for now.

$$k = \frac{P_c}{T^\delta * \left(1 + \left(\frac{V}{V_b}\right)^\alpha * \left(\frac{T}{T_b}\right)^\beta\right) * V^2}$$

$$k = \frac{0.00324}{1^{-0.65} * \left(1 + \left(\frac{1.25}{1.25}\right)^1 * \left(\frac{1}{5.2}\right)^{2.5}\right) * 1.25^2}$$

$$k = 0.002028$$

The constant  $\alpha$  can then be solved using a different point having a different voltage. In actuality, I derived the equation in SPICE and adjusted the parameters until there was a good fit. It is hard to get the constant  $\delta = -0.65$  right from the graph, as the asymptote is hard to visualize and graph, but it was quite easy to zero in on it in SPICE.

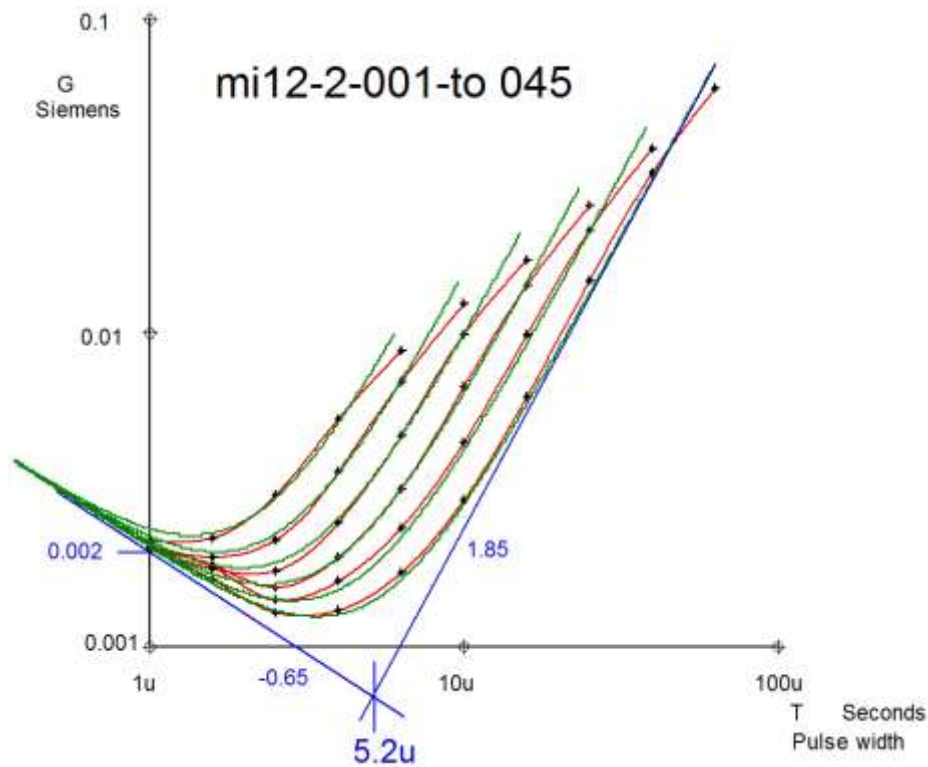
Note, this is NOT a SPICE model to be used for circuit modeling. However, SPICE is very useful for solving equations expressed in behavioral voltage or current sources and for plotting the results. Actual data can be imported easily for comparison.

The Appendix shows some of the intuitive steps used in the derivation.

# Appendix

## Conductance curves

The first step was to use the data for the Herbert curve above, but divide through by the voltage squared, resulting in the curves below.



The curves are loosely defined as “Conductance curves,” because they were derived from the following reasoning.

$$P = \frac{V^2}{R}$$
$$\frac{1}{R} = \frac{P}{V^2} = G$$

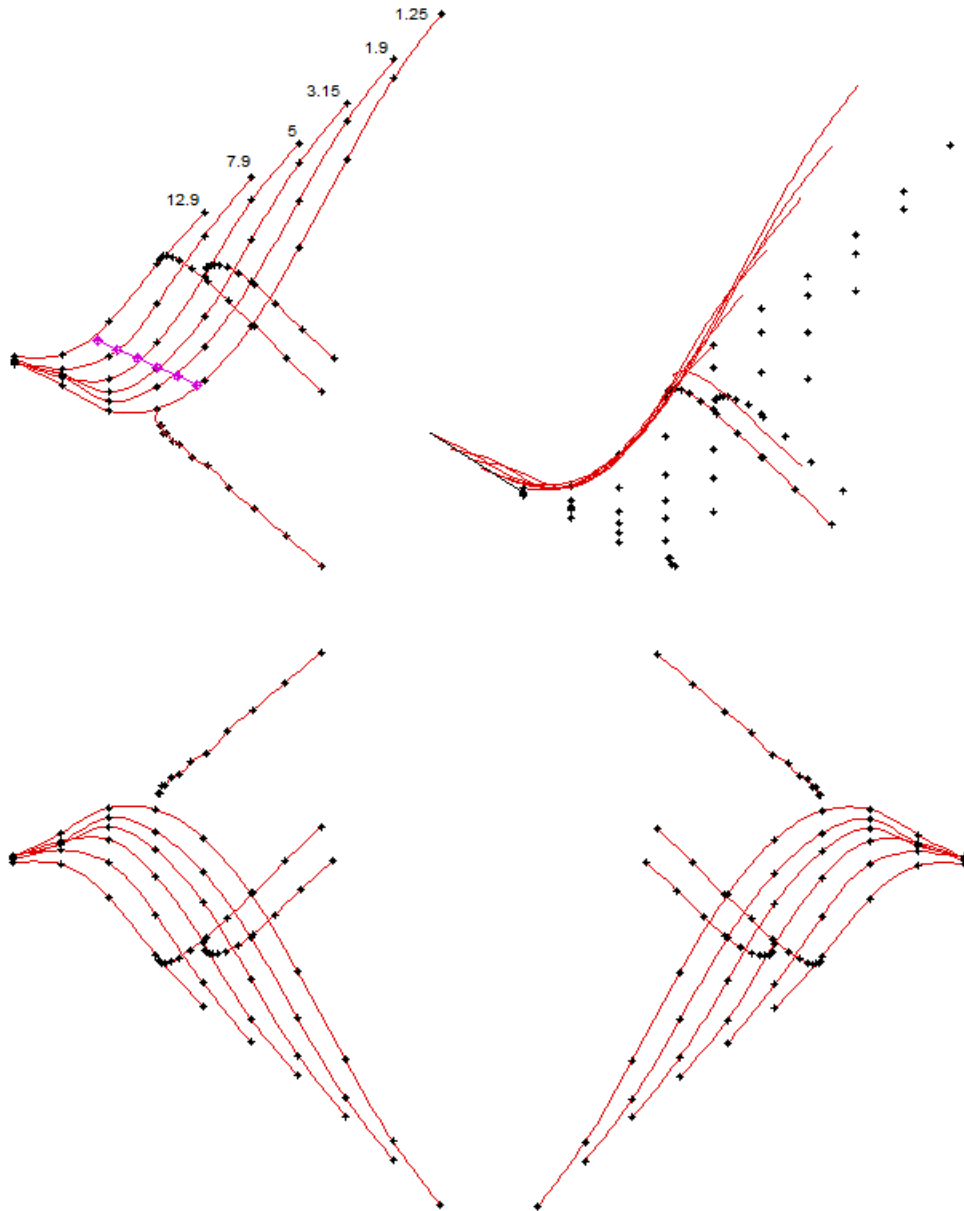
The idea was to write an equation for the resistance as a function of frequency, a familiar operation from feedback theory. Because conductance fell out of the first operation, dividing the core loss  $P$  through by  $V^2$ , I left it that way. Also, the curves were in the form of Herbert curves with the x-axis being  $T$ , not frequency, that was also left that way.

Note that the intercept of the first asymptote and  $T=1 \mu\text{s}$  gives a direct reading of the constant  $k = 0.002$ .

## CAD Doodles

Below are some curve families that I called “CAD doodles.” The upper left is the conductance curves above as first imported into CAD. The drooping curves are from the expand data, and the magenta curve is exploring the spacing of the voltage curves.

On a log-log plot, there are some interesting relationships. Reciprocals are mirror images, so the lower left curves are the resistance equivalent, and the lower right curves are resistance with increasing frequency as the x-axis.



The curves on the upper right showed that the curves nest fairly well, leading to an expectation that a conductance curve for one voltage can be scaled to other voltages.

## SPICE model

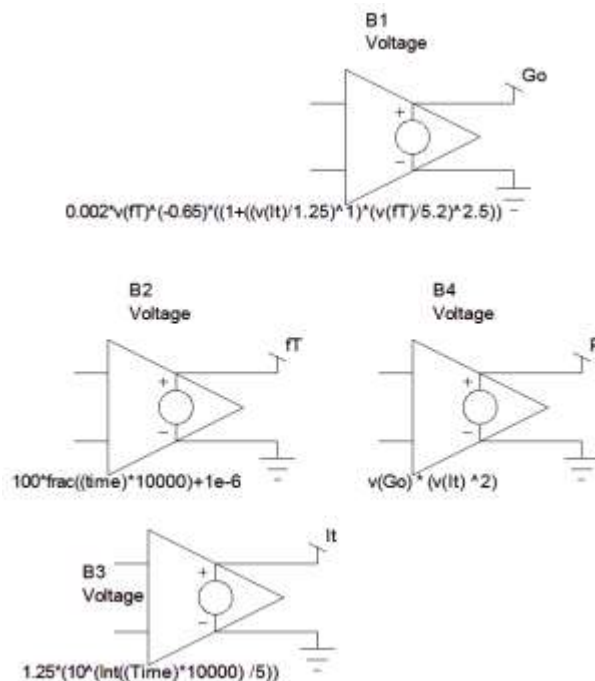
NOTE: This is NOT a SPICE model to be used for circuit modeling. However, SPICE is very useful for solving equations expressed in behavioral voltage or current sources and for plotting the results. Actual data can be imported easily for comparison.

Since I was working with the conductance curves, the starting point was:

$$G_c = k * T^\delta * \left(1 + \left(\frac{T}{T_b}\right)^\beta\right)$$

This was used to get the shape of the graph, using the lowest (1.25 V) curve. Then a voltage relationship was proposed

$$G_c = k * T^\delta * \left(1 + \left(\frac{V}{V_b}\right)^\alpha * \left(\frac{T}{T_b}\right)^\beta\right)$$

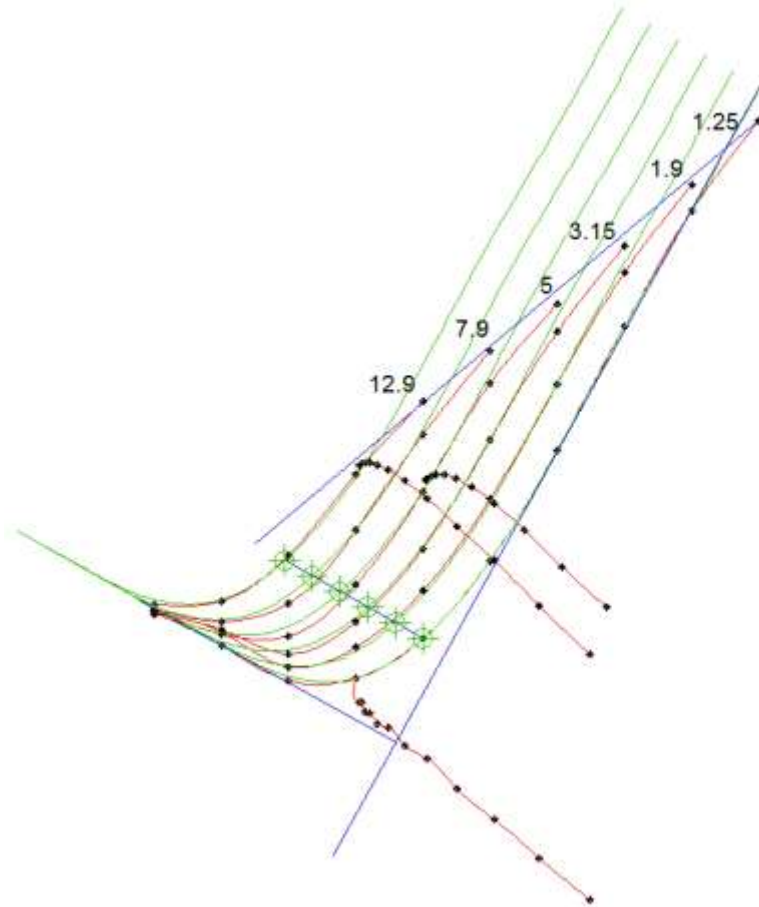


In SPICE, the expression for the conductance  $G_c$  can be written as

$$G_c = 0.002 * v(fT)^{-0.65} * \left(1 + \left(\frac{v(It)}{1.25}\right)^1 * \left(\frac{v(fT)}{5.2}\right)^{2.5}\right)$$

Once we have a solution for  $G_c$ , core loss  $P_c$  is just the conductance  $G_c$  times the voltage squared

$$P_c = v(G_c) * v(fT)^2$$



In SPICE, the function “time” is just what it seems, so in a behavioral model, “time” gives a voltage ramp incrementing with time. Taking the “frac” of time gives a repeated time ramps resetting at each second increment. Multiplying the time by 10,000 before taking the fractional part gives a repeating ramp of 100  $\mu$ s duration, and multiplying the frac expression by 100 gives a time reference fT where T ramps from 0 to 100  $\mu$ s, then repeats.

By the same logic, taking the “int” of time gives steps incrementing by 1 each second. Multiplying by 10,000 before taking the integer gives steps at 100  $\mu$ s increments. Dividing by 5, the steps become increments of 0.2. By using that expression as the exponent of 10, the steps become logarithmic. Initially, the result is 0, and  $10^0 = 1$ . Multiplying by 1.25, the first step is 1.25 V, and the output increments in the correct sequence to match the voltages used in the data, 1.25; 1.9; 3.14, 4.97; 7.99; and 12.5.

The result can be graphed using the SPICE oscillograph function, with v(fT) as the x-axis and Go as the Y a-axis. Use the log-log plot. The graph can then be copied into CAD. With careful scaling, the curves can be fitted to the data as shown above.

The exponent of the voltage function,  $\alpha$ , gives a good fit when  $\alpha = 1$ , so maybe the exponent is not necessary. With just one set of data processed, it seems prudent to leave it as a constant to be determined.