Optimization of Power Magnetics Design

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Presented by

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Magnetics design optimization is defined, performance criteria are identified, the basic conflict in magnetics design identified, maximum core transfer power constraints are related to optimal turns, optimal ripple factor is related to core material, and other optimizations are listed.
Magnetics Research Directions

• Higher-frequency magnetics
  Charles Sullivan, Dartmouth
  David Perreault, MIT

• More accurate magnetics models
  Chema Molina, Frenetics AI, Madrid, Spain
  Bryce Hesterman, Utah State U.

• Magnetics design optimization
  Innovatia, Cayo, Belize
Definitions

• *Waveform*: electrical function of time
  - \( v(t), i(t), \) sometimes \( P(t) \)
  - \( x(t) \) is \( v(t) \) or \( i(t) \)
  - Waveforms decompose into a sum of average value \( \bar{x} \) or \( X \) and varying or ripple value \( x_\sim \):
    \[
    x = X + x_\sim
    \]

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>( \bar{x} )</td>
<td>average ( x )</td>
</tr>
<tr>
<td>( \hat{x} )</td>
<td>peak (maximum) ( x )</td>
</tr>
<tr>
<td>( \tilde{x} )</td>
<td>rms value of ( x )</td>
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More Definitions

- **Waveshape**: a waveform without scaling

  \[
  \text{waveshape of } x(t) = \frac{x(t/T)}{\hat{x}}
  \]

- **Transductor**: the *structural* name for a multiple-winding magnetics component

  *Transformer* and *coupled inductor* name behaviors, not structure

  No name in magnetics for structure, only behavior
Optimization

Q: What is optimization?

A: Finding the best solution to a design problem with conflicting criteria.  

*Criteria* are what decisions are based on.  

Optimization criteria are quantified as  

*performance parameters*
Performance Parameters

Efficiency = power out/power in

Form factor = \( \kappa = \frac{\tilde{x}}{\hat{x}} \)

Crest factor = \( \chi = \frac{\hat{x}}{\tilde{x}} \) (inverters)

Utilization = \( U = \frac{\bar{x}}{\hat{x}} \) (components)

Ripple factor = \( \gamma = \frac{\Delta x / 2}{\bar{x}} = \frac{\hat{x}}{\bar{x}} \)

DCM: \( \gamma > 1 \); CCM: \( \gamma \leq 1 \)
Ideal Performance
Parameter Values

Ideally, $\kappa = 1$, $\chi = 1$, $U = 1$ and
\[
\Delta x = 0, \quad \bar{x} = 1 \quad \Rightarrow \gamma = 0
\]

Q: What is the ideal waveform?
A: A constant waveform. ($\gamma = 0$)

Waveform average component is an idealization of a constant waveform

Power transfer through reactances:
\[
\Delta x \neq 0: \quad v = L \cdot \frac{di}{dt} \approx L \cdot \frac{\Delta i}{\Delta t}, \quad \Delta i \neq 0 \text{ A}
\]
Magnetics Design
Basic Conflict

Ripple components of waveforms
• detract from the ideal, yet
• are essential for power transfer

The Power Transfer Equation:
\[ \overline{P} = \Delta W \cdot f_s \]

Linear energy = \[ W_L = \frac{1}{2} \cdot L \cdot \hat{i}^2 \]

\[ \Rightarrow \Delta W_L = L \cdot I \cdot \Delta i \]
Power Transfer Equation

\[ \overline{P}_{xfr} = (L \cdot I \cdot \Delta i) \cdot f_s \]

Core power transfer: \( \Delta i \text{ and } I \neq 0 \) A

Conflict:
For performance, minimize \( \Delta i \)
For power transfer, maximize \( I \)

\( \Rightarrow \) Optimal \( \gamma \approx 0 \) \( \Rightarrow \)

Small-ripple approximation:
\[ \Delta x \ll X \text{ or } \Delta i \ll \bar{i} \]
Maximum Core Utilization

Design Goal: Maximum core transfer-power density \( \Rightarrow \) core \( U = 1 \)

Linear transfer energy each cycle =

\[
\Delta W_L = [\Delta B \cdot \bar{H}] \cdot V
\]

Linear energy density =

\[
\Delta w_L = \frac{\Delta W_L}{V} = \Delta B \cdot \bar{H}
\]

\( \Rightarrow \) Transfer power proportional to \( f_s \):

\[
P_{xfr} = \Delta W \cdot f_s \approx \Delta W_L \cdot f_s
\]
Core Limitations

• Temperature ⇒ power loss ⇒ ΔB
• Saturation ⇒ field intensity \( \bar{H} \)

Magnetic-field ripple =

\[ \Delta B = \text{twice amplitude} \quad \hat{B}_\sim = \Delta B / 2 \]

Core power-loss density \( \bar{p}_c(\hat{B}_\sim, f_s) \)

At magnetic op-pt \( \bar{H} < H_{sat} \)

Related to average field current \( \bar{N}i \)

Saturation factor

\[ k_{sat} = \frac{L(I)}{L(0 \ A)} \leq 1 \]
Maximum Core Utilization

⇒ Core fully utilized when driven to both limits ⇒ maximum transfer power density

Maximum power-transfer conditions:

Core is driven with as large of a

○ $\Delta B$ as the thermal limit allows
  → limited by $\overline{\rho}_c(\hat{B}_\sim, f_s)$

○ $\overline{H}, N\overline{i}$ as saturation allows
  → limited by $k_{sat}$

⇒ $\Delta w_L = \Delta B \cdot \overline{H}$ is maximized
Turns as Design Parameter

Central magnetic design parameter: turns $N$

Maximum core utilization determines turns limits:

Too few turns over-heats core

$$N \geq N_\lambda = \frac{\Delta \lambda}{\Delta \phi(\bar{P}_c)} \quad \Delta \lambda = \text{circuit flux}$$

Too many turns over-saturates core

$$N \leq N_i = \frac{\bar{N} \bar{i}}{\bar{i}} \quad \bar{N} \bar{i} = \text{average field current}$$

Maximum turns that fit winding window $N_w$

$N_w$ is also a turns limitation → allowable current density
Turns as Design Parameter

Design range of $N$ is bounded by these limits:

$$N_{\lambda} \leq N \leq \min\{N_i, N_w\}$$

- Core fully utilized when $N_{\lambda} = N_i$:

$$N_{opt} = N_{\lambda} = \frac{V_p \cdot t_{on}}{(2 \cdot \hat{B}_\infty) \cdot A} = N_i = \frac{N \tilde{i}}{I_p}$$

Circuit flux $\Delta \lambda = V_p \cdot t_{on}$, $I_p =$ primary on-time current

$$\frac{V_p \cdot t_{on}}{(2 \cdot \hat{B}_\infty) \cdot A} = \frac{N \tilde{i}}{I_p}$$

Solve for primary-winding on-time power amplitude:

$$P_p = V_p \cdot I_p = [(2 \cdot \hat{B}_\infty) \cdot A \cdot (f_s / D)] \cdot N \tilde{i}$$

$$\bar{P}_p = D \cdot V_p \cdot I_p = \{[(2 \cdot \hat{B}_\infty) \cdot A \cdot N \tilde{i}] \cdot f_s = \Delta W_L \cdot f_s$$

$$1/t_{on} = f_s / D, \ D = \text{duty-ratio (duty cycle)}$$
Ripple Factor & Power

Ripple factor is related to max power transfer:

\[
\gamma = \frac{\bar{i}}{i} = \frac{\Delta i}{2I} = \frac{\hat{H}}{\hat{H}}
\]

\[
\gamma_{opt} = \frac{\hat{H}}{\hat{H}} = \frac{\hat{B}}{\mu \cdot A} \cdot \frac{A}{N^2 i / l} = \frac{\hat{B} \cdot A}{\mathbf{L} \cdot N^2 i} = \frac{\phi}{\phi}
\]

Field inductance = \( \mathbf{L} = k_{sat} \cdot \mathbf{L}_0 = \mu \cdot A / l \)

Maximum transfer power subject to the condition that circuit \( \gamma = \gamma_{opt} \) of core

Current waveform \( \gamma = \gamma_{opt} \) for max power, depends on core material \( \Rightarrow \)

waveform \( \leftrightarrow \) core material match

\( \gamma_{opt} \) should be in core catalogs
Other Optimizations

• Magnetic switching frequency, $f_{MAX}$ where $d(\text{loss}) = d(\text{xfr power})$
  - $f_{MAX}$ needed in core catalogs

• Minimum eddy-current resistance subject to
  - given layers (in textbooks)
  - given winding area
  - given strands per bundle turn

• Maximum winding transfer efficiency over a current range

• A shape-based thermal model
Other Optimizations

- An asymptotic saturation model
  Saturation curves approximated by line segment in saturation region:

\[ \log\left( \frac{H_T}{H_0} \right) \text{ parameter} = \text{decades of range of core saturation region} \]

\[ k_{sat}(L_{max}) \text{ determined by core material properties at current } I \text{ operating-point} \]

\[ k_{sat} \text{ chosen optimally instead of arbitrarily} \]

- Accurate toroid winding length
References

*Power Magnetics Design Optimization*, Innovatia

PDF book available free to PSMA HF Power Magnetics Workshop attendees; request via email: dennisf@innovatia.com

Nearly 100 magnetics design optimization articles at how2power.com