On Size and Magnetics:
Why Small Efficient Power Inductors Are Rare

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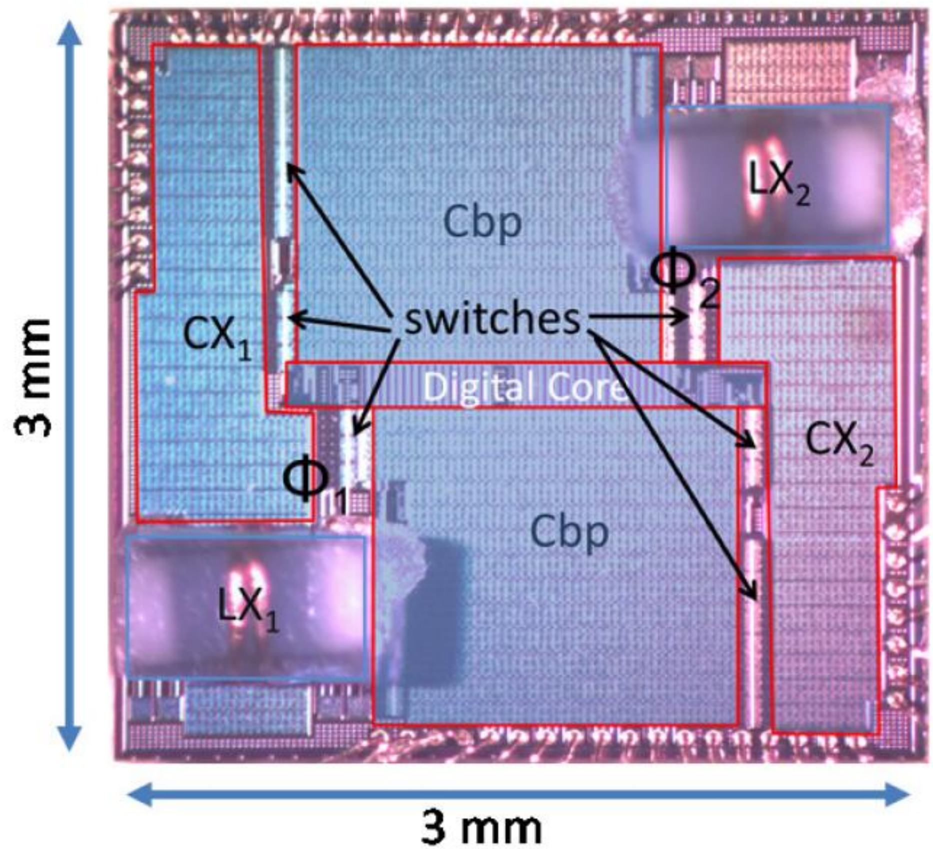
Basic power electronics elements: Switches, capacitors and inductors

- Switches and capacitors:
  - Routinely fabricated on-chip.
  - Routinely made by combining thousands of parallel cells.
- Inductors and other magnetics:
  - Why not on-chip cellular approach?

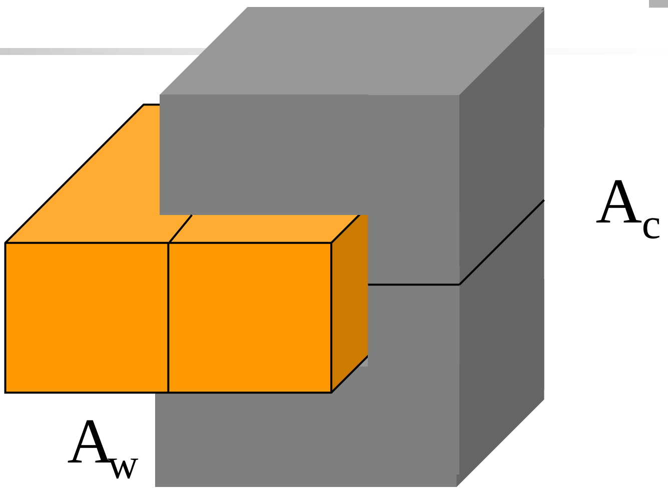


Resonant switched capacitor converter, Kesarwani, Sangwan and Stauth (Dartmouth), Trans. Pow El. 2015

Why not arrays of on-chip cellular inductors?

- Process and materials?
  - Hard, but feasible.
- Scaling?
  - Can 100, 1000, or 1,000,000 inductor cells in parallel or series be used to make one good high-power inductor?
  - How does performance scale with size?
    - Does a 1% scale inductor, handling 1% of full power work as well as a single full size, full power inductor?
  - Can we make small high performance inductors?

A simple case



- Fixed frequency.
- B_0 limited by core loss.
- J_0 limited by winding loss.
- Find power handling VA:

$$VA = V \cdot I \propto (N f B_0 A_c)\left(\frac{J_0 A_w}{N}\right) = f(B_0 J_0)(A_c A_w)$$

- Number of turns cancels out—scales impedance without affecting power capability or loss.
- Scale all linear dimensions by a factor ϵ
  - Areas scale by a factor ϵ^2
  - VA scales by a factor ϵ^4 … faster than volume (ϵ^3)!

Simple case: implications

Simple case = fixed loss densities mean fixed B_0, J_0

- $VA \propto \epsilon^4$
- Power density = $(VA)/\text{volume} \propto \epsilon^4/\epsilon^3 = \epsilon \propto \sqrt[3]{\text{volume}}$
- Power density is better in large sizes;
  worse in small sizes.
- This is the most fundamental/basic reason why 100 small inductors are worse than one big one.

Simple case: implications (continued)

- Fixed loss densities mean loss $\propto$ volume $\propto \epsilon^3$
- Loss fraction (loss/VA) goes as $\epsilon^3/\epsilon^4 = \epsilon^{-1}$
  - Loss fraction is inversely proportional to linear scaling factor.
- (From last page: power density $\propto \epsilon$)
- Both power density and efficiency are better in large sizes and worse in small sizes.

Example:
1 full-size L vs. 100 small Ls

- 100 small inductors can be:
  - In series, each L'=L/100.
  - In parallel, each 1/100 full current, but 100X L value.
  - Network of 10 in series/10 in parallel, same L value.
  - In each, energy storage and VA are reduced by 100X
- With our assumptions, these options are equivalent.
  - Adjust N to obtain value needed; doesn't change loss or power handling (VA).
- Can they be tiny?

Example:
1 full-size L vs. 100 small Ls

- 100 small inductors are not so small:
  - At least 3.16 X total volume
  - At least 3.16 X total loss

Other considerations

- Constant loss density might not make sense:
  - Cooling limited by surface area, not volume
  - Efficiency might be real limit, not cooling (especially at small sizes).
- High frequency winding loss:
  - Depends on size vs. skin depth δ.
  - AC resistance problems are worse for large sizes.

Constraint on: loss per unit surface area

- Steinmetz model for core loss: $P_v = kB^\beta, \quad 2 < \beta < 3$ **β = 2.5**
  - Power law for flux density is very accurate even though a power law for frequency dependence is not.

Constraint:
loss $\propto \epsilon^2$

Models:
$P_v = kB^\beta$
$P_v = \frac{\rho}{2} J^2$

Algebra

$$VA \propto \epsilon^{3.5-1/\beta} \approx \epsilon^{3.1} \qquad \frac{P_{loss}}{VA} = \epsilon^{-1.5+1/\beta} \approx \epsilon^{-1.1}$$

- Same trends: power density and efficiency are better in larger sizes.

Constraint on: efficiency

- Same models.

Constraint: Fixed loss fraction

Models:

$$P_v = kB^\beta$$

$$P_v = \frac{\rho}{2} J^2$$

Algebra

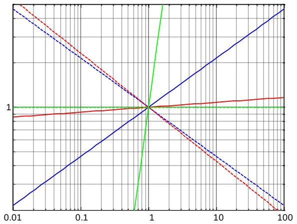
$$VA \propto \epsilon^{3+\frac{2\beta}{\beta-2}} \approx \epsilon^{13}$$

- Maintaining the same efficiency in small sizes is very hard—power density plummets.

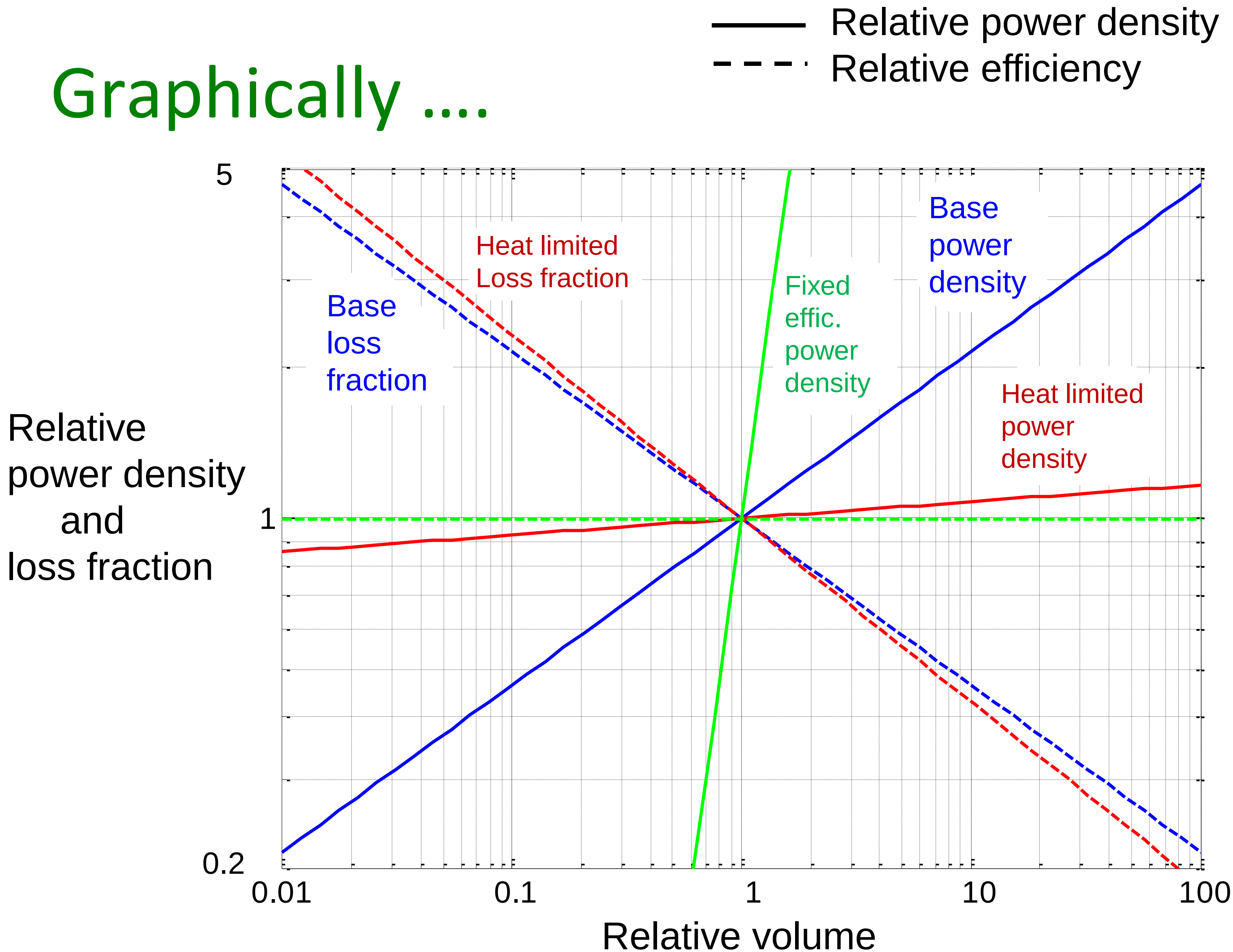
Summary so far ..

Constraint	VA	VA/Volume	$P_{\text{loss}}/\text{VA}$
		Low Frequency	
Loss density	ϵ^4	ϵ	ϵ^{-1}
Heat flux	$\approx \epsilon^{3.1}$	$\approx \epsilon^{0.1}$	$\approx \epsilon^{-1.1}$
Efficiency	$\approx \epsilon^{13}$	$\approx \epsilon^{10}$	Constant

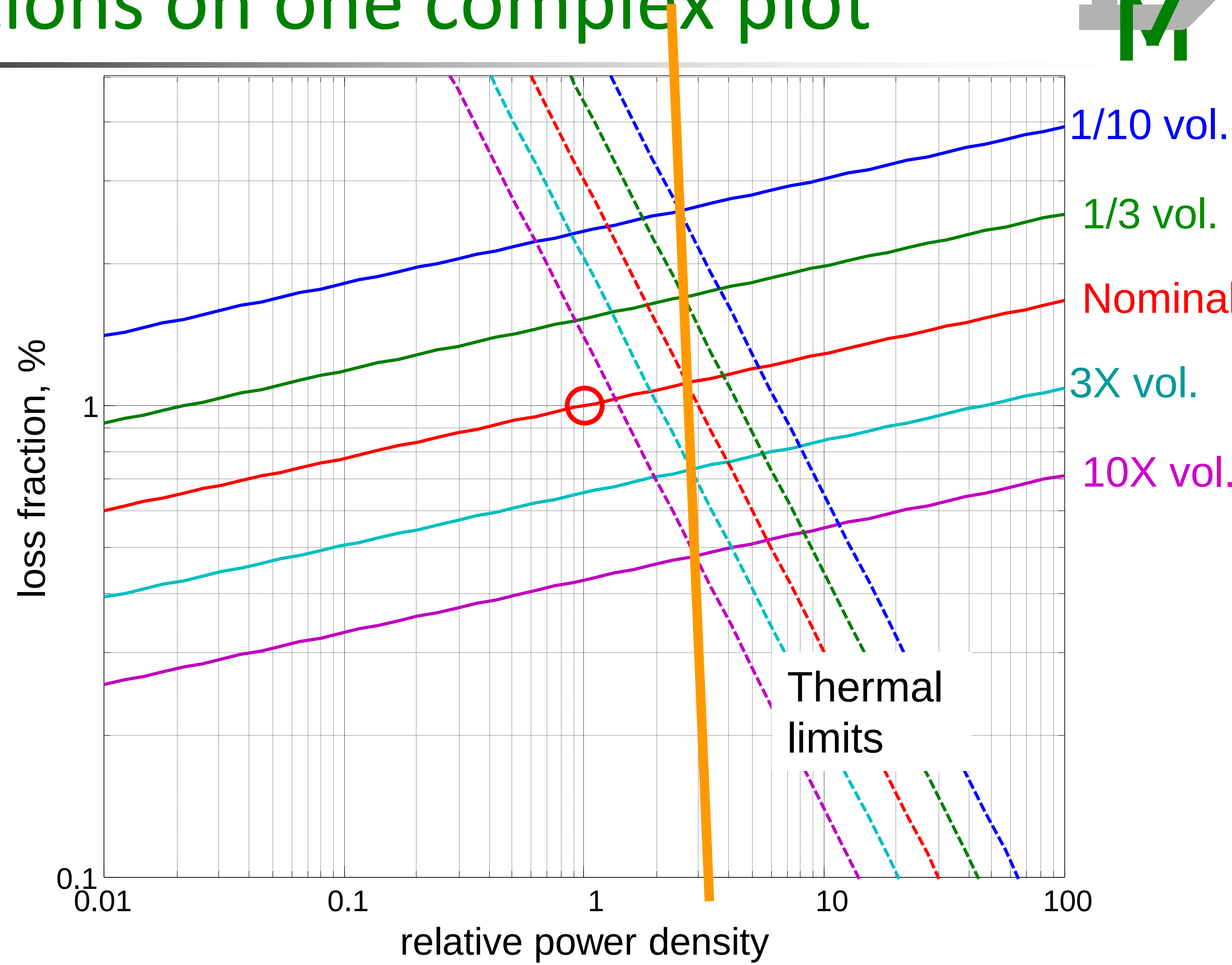
- Or, graphically…



Graphically ….



All options on one complex plot



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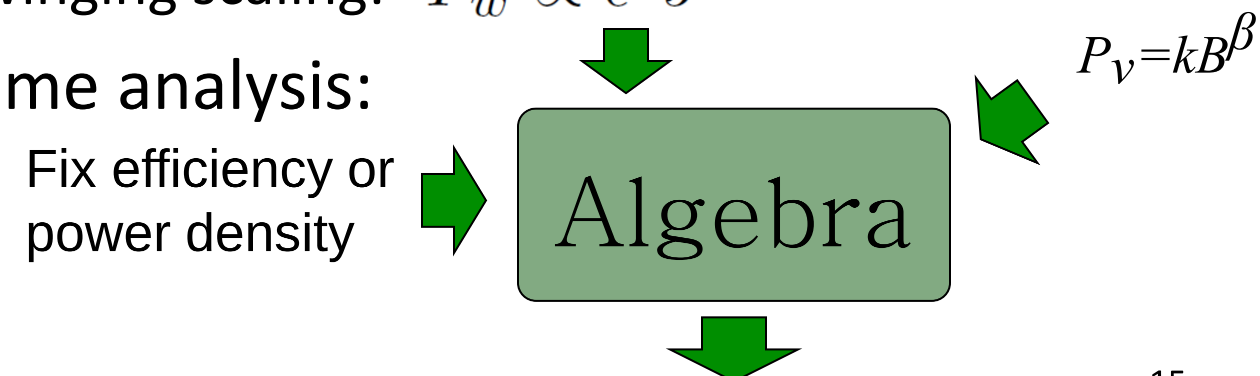
With high-frequency winding loss

- Straight scaling would lead to bad designs
- Instead: optimize winding design for each scale.
  - Various optimization constraints possible.
  - Two lead to the same scaling:
    - Fixed number of layers (including single layer) with optimized layer thickness
    - Optimized number of layers with fixed layer thickness.
    - Resulting winging scaling: $P_w \propto \epsilon^2 J^2$
- Put this into same analysis:

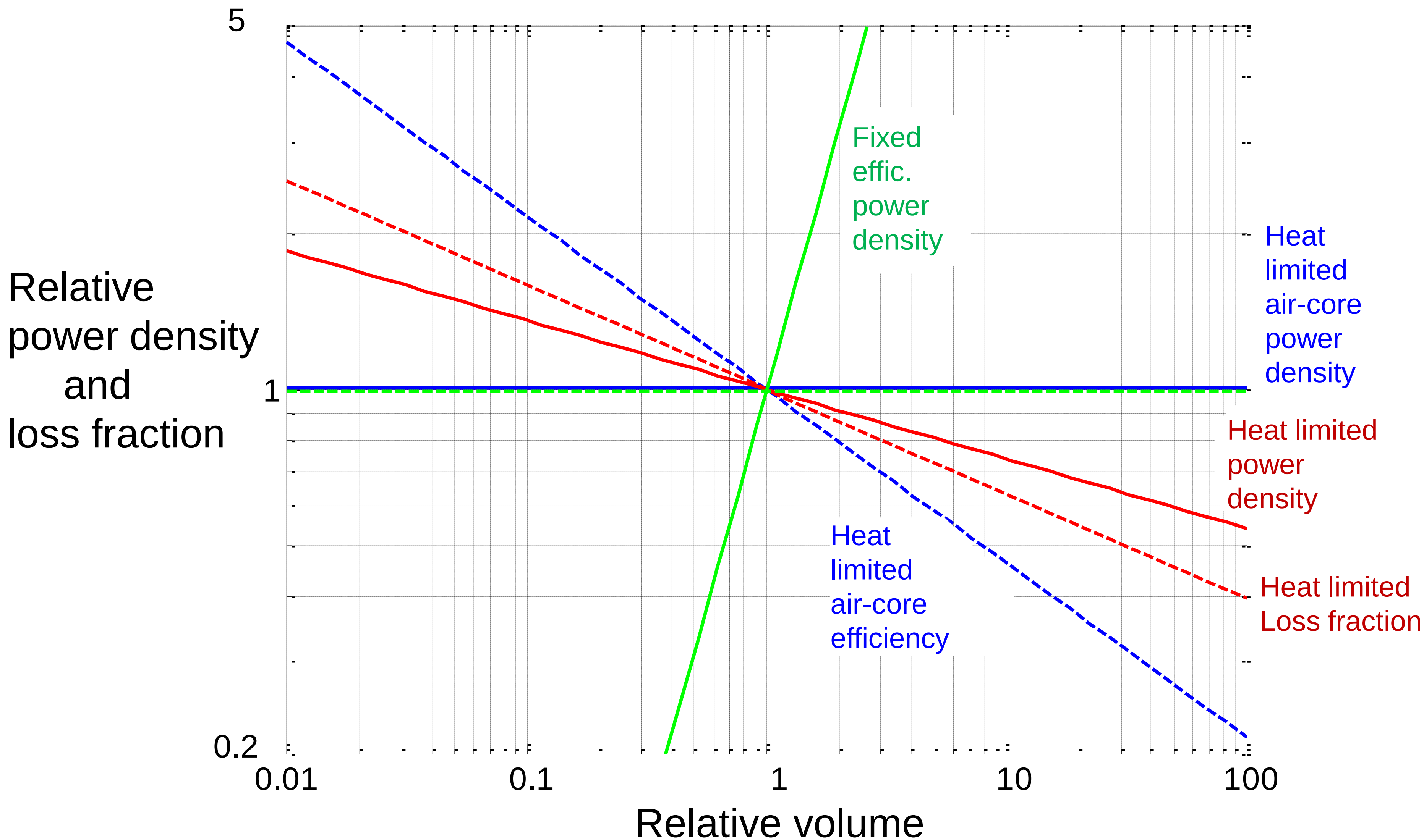


Complete results

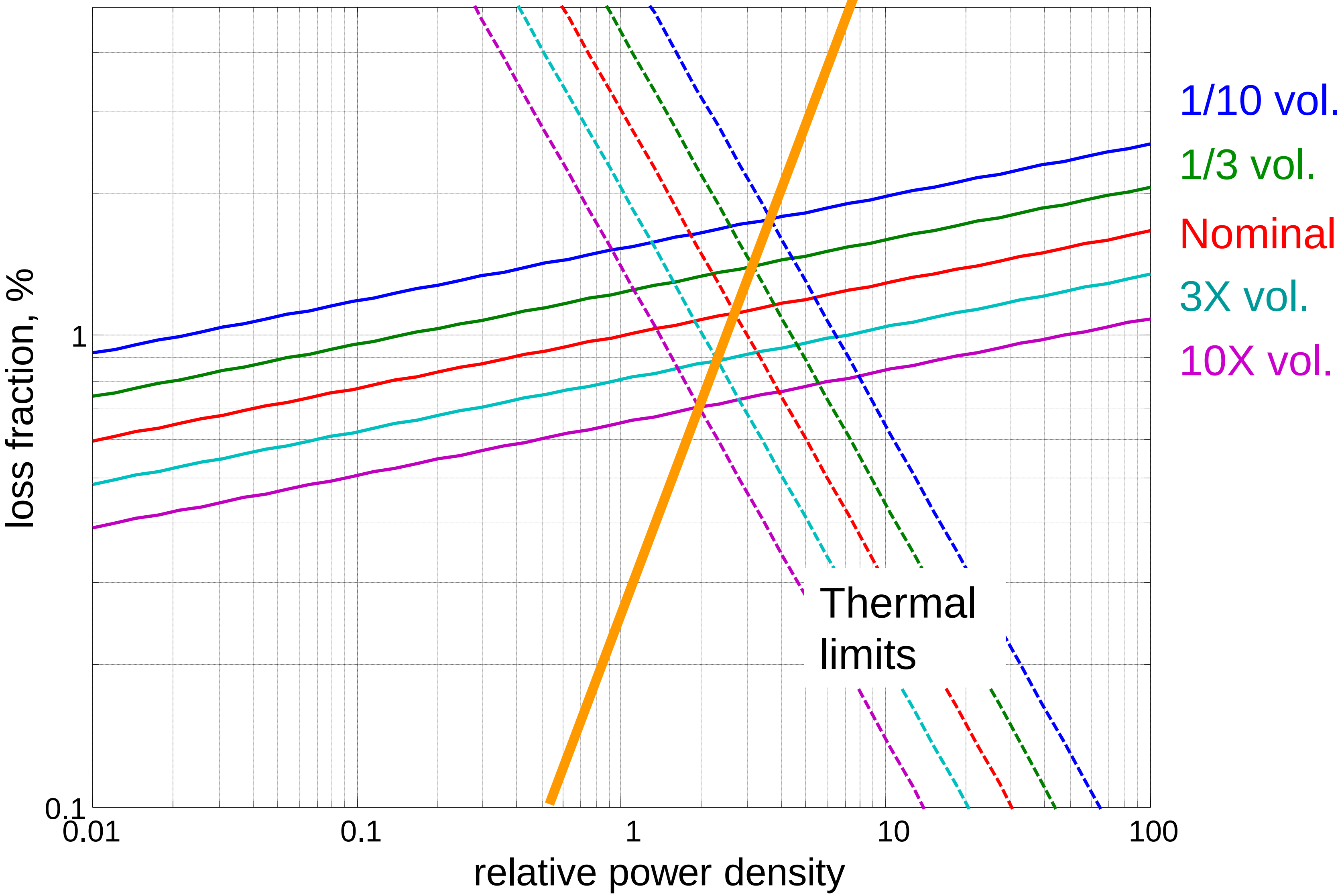
- Efficiency and power density usually better with large sizes.
- Only exception: Heat-flux-limited high-frequency windings have slightly better power density at small sizes.
  - But efficiency is worse at small sizes.
  - Poor efficiency means efficiency is real constraint, and then power density drops fast.
  - Thermal situation for an array of components can be worse than for one.

Constraint	VA	VA/Volume	P_{loss}/VA
Low Frequency			
Loss density	ϵ^4	ϵ	ϵ^{-1}
Heat flux	$\approx \epsilon^{3.1}$	$\approx \epsilon^{0.1}$	$\approx \epsilon^{-1.1}$
Efficiency	$\approx \epsilon^{13}$	$\approx \epsilon^{10}$	Constant
High Frequency			
Heat flux	$\approx \epsilon^{2.6}$	$\approx \epsilon^{-0.4}$	$\approx \epsilon^{-0.6}$
Efficiency	$\approx \epsilon^{8}$	$\approx \epsilon^{5}$	Constant
High Frequency Air-Core [3]			
Heat flux	ϵ^3	Constant	ϵ^{-1}

Graphically …. with high-frequency winding loss effects

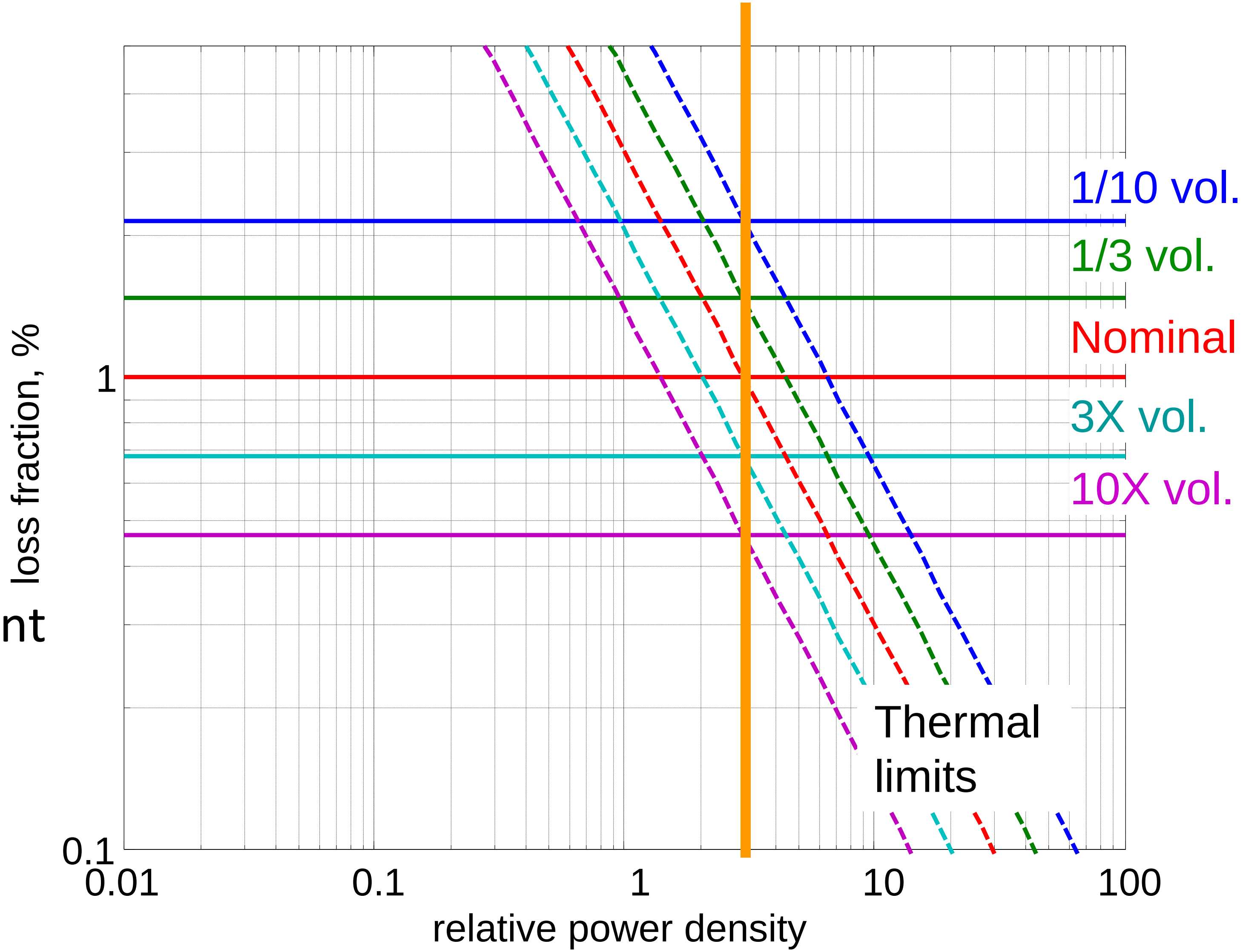


The complex chart again, with HF winding loss



One more time, for air core

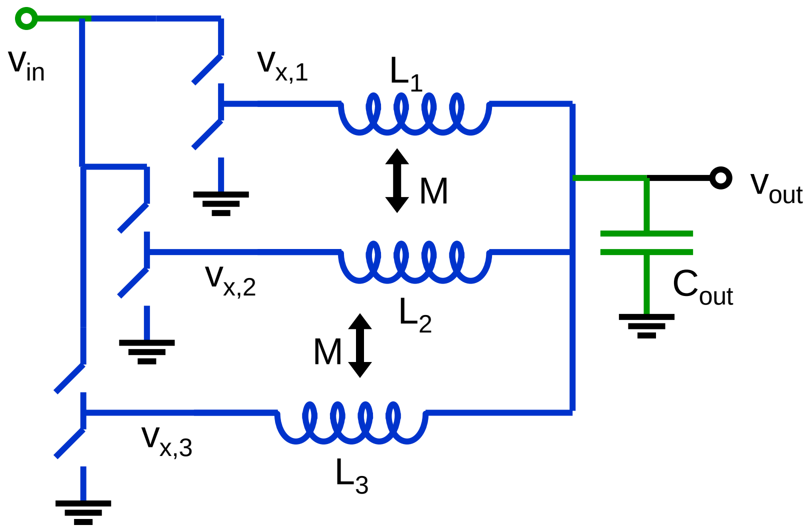
- Scale determines efficiency.
- Power density thermally limited (independent of scale)

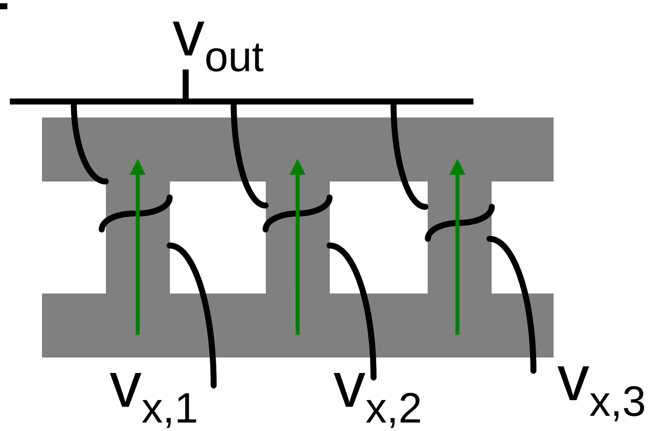


Implications

- A single inductor or transformer for the full power is better than combining many small ones.
  - Better power density
  - Better efficiency
- Drives need for custom magnetics.
- Also motivates designs with a shared single core:
  - Single inductor with complex circuit
    (e.g. various approaches with switched capacitors).
  - Coupled-inductor designs.
  - Multi-channel transformer-coupled converters with separate windings on a single core (ref. 14 of presentation).

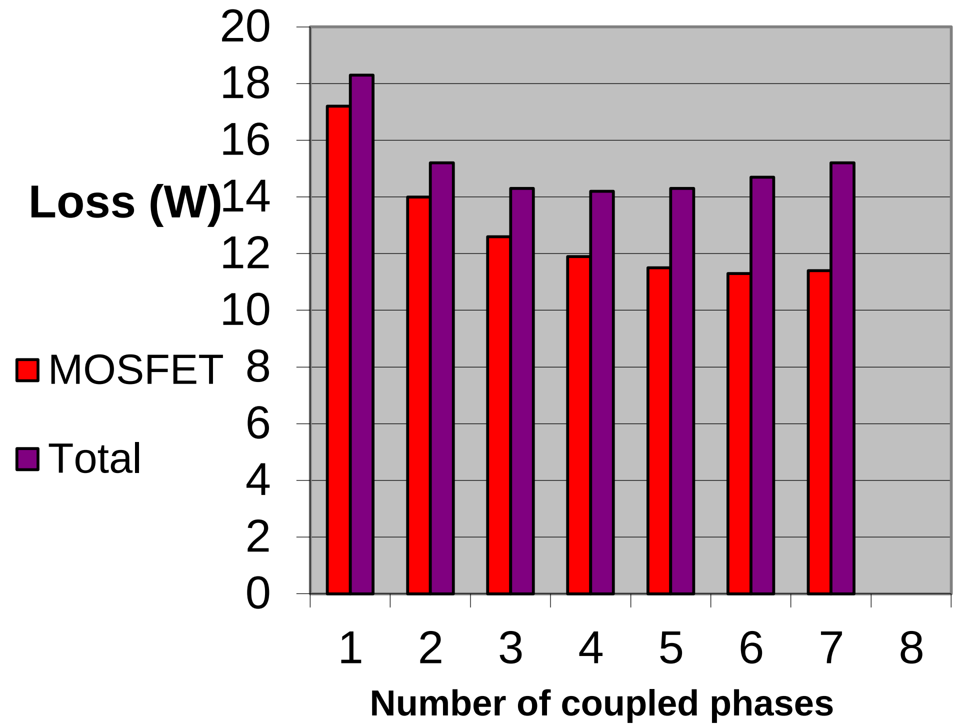
Coupled inductor buck optimization example (2002)





No coupling benefit

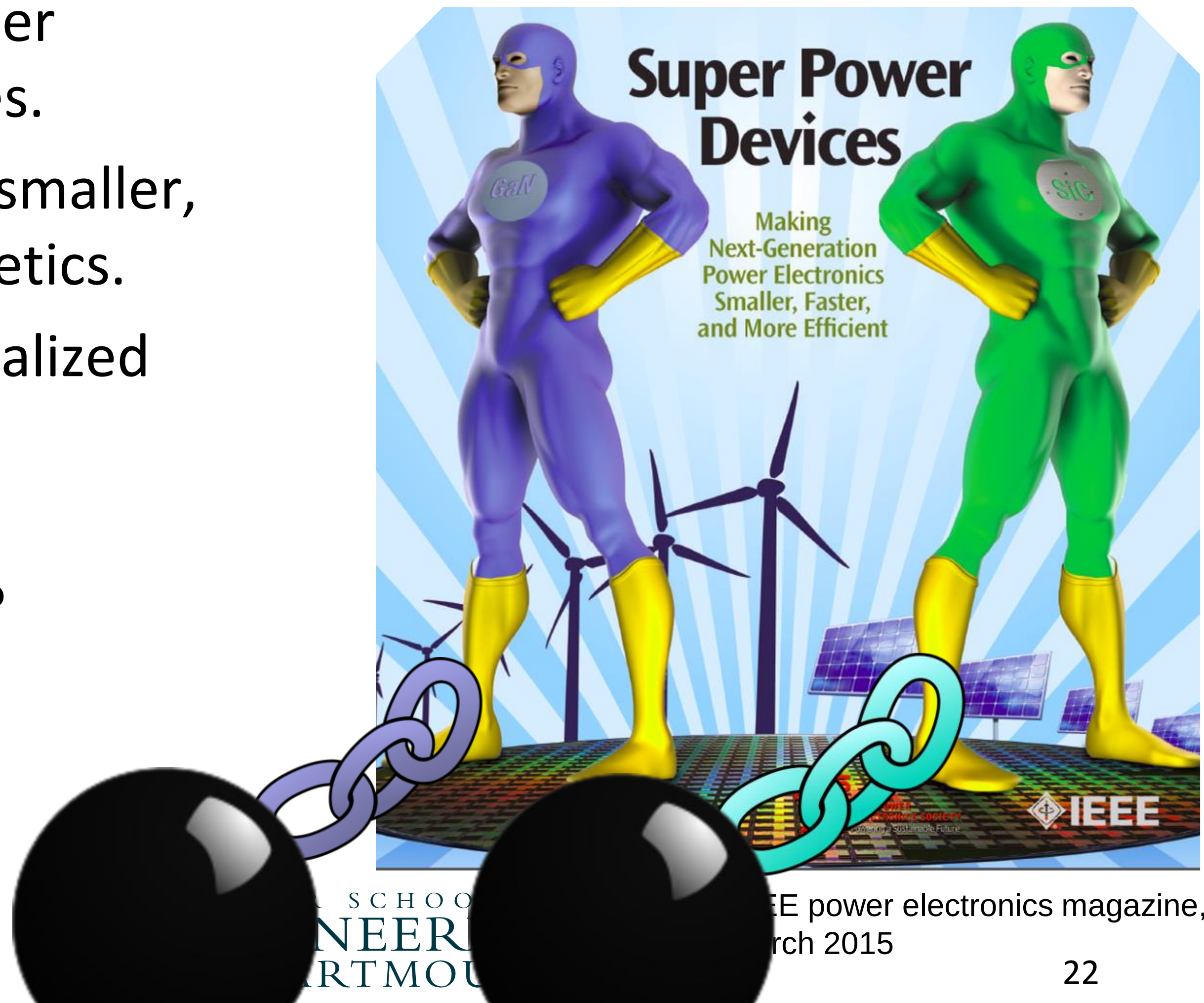
Each L is too small



Comparison of designs, each optimized for a specific number of phases

Modular/cellular approach won't lead to tiny power magnetics. What will?

- New semiconductors emerging: GaN and SiC power devices.
- Can allow much higher switching frequencies.
- Theoretically allows smaller, more efficient magnetics.
  - But can this be realized in practice?
    - Windings?
    - Core materials?



Credit to Jelena Popovic and Dragan Maksimovic for the ball and chain analogy

E power electronics magazine,
rch 2015

Bonus content: scaling vs. f

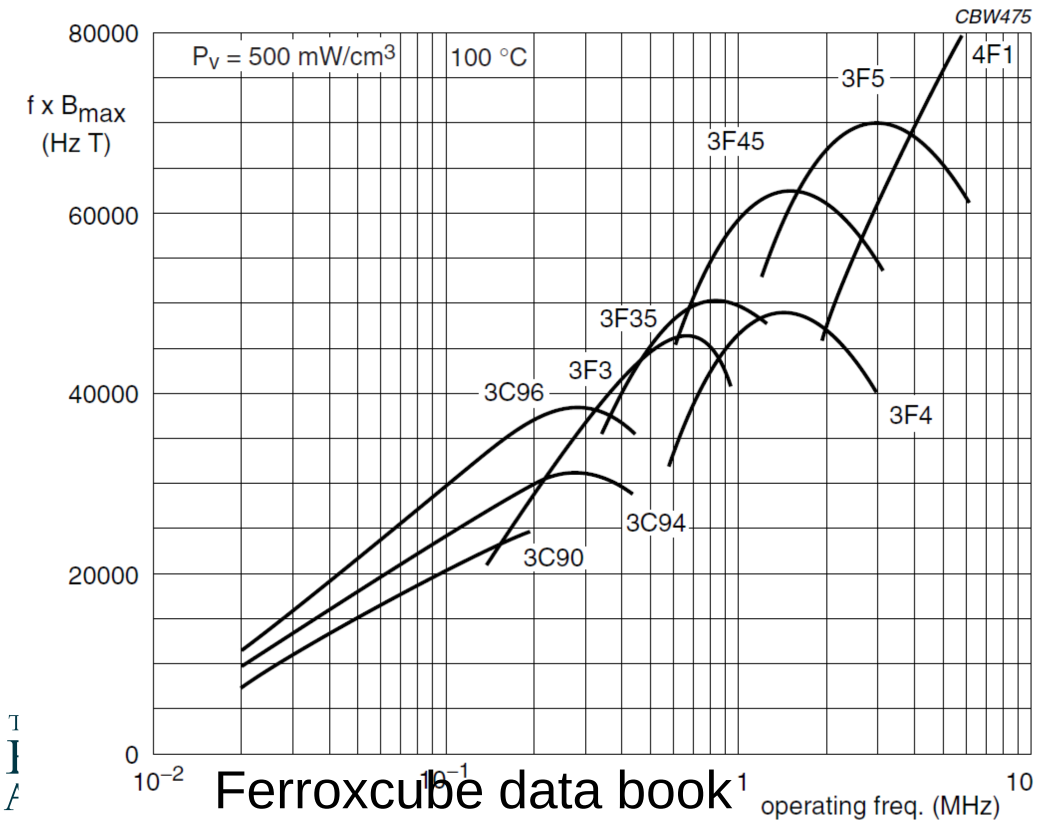
- Air-core inductors: Loss fraction $\propto \frac{1}{\sqrt{f}}$ for typical windings (e.g., single-layers)
- Magnetic-core inductors:
  - Depends on frequency dependence of core loss.
  - Frequency dependence of core loss is not well behaved—need actual data.
  - Also depends on winding type.

Figure of merit for cores: performance factor

- Before, we had:

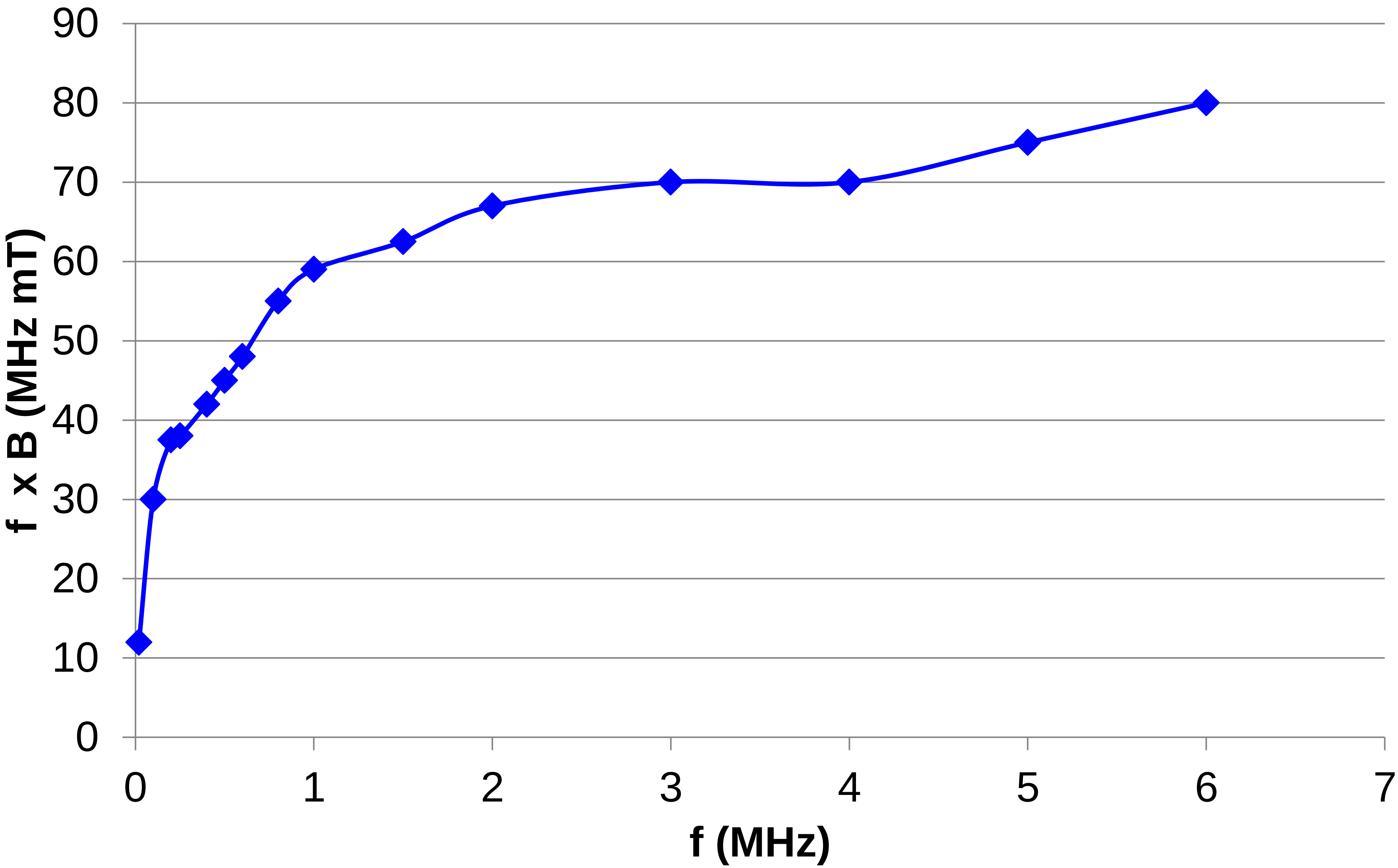
$$VA = V \cdot I \propto (N f B_0 A_c)\left(\frac{J_0 A_w}{N}\right) = (f(B_0)J_0)(A_c A_w)$$

- Standard figure of merit: "Performance factor" $B_0 \cdot f$
- For each frequency, *f*, choose flux density B_0 based on maximum tolerable loss.



Ferroxcube data book

Performance factor, linear scale



Some gain at high frequency—is it worth the pain?

- Other things get more difficult—board layout, skin effect and proximity effect in windings …
  - Can capture effect of high-frequency winding losses in a *modified performance factor.*

Modified performance factor

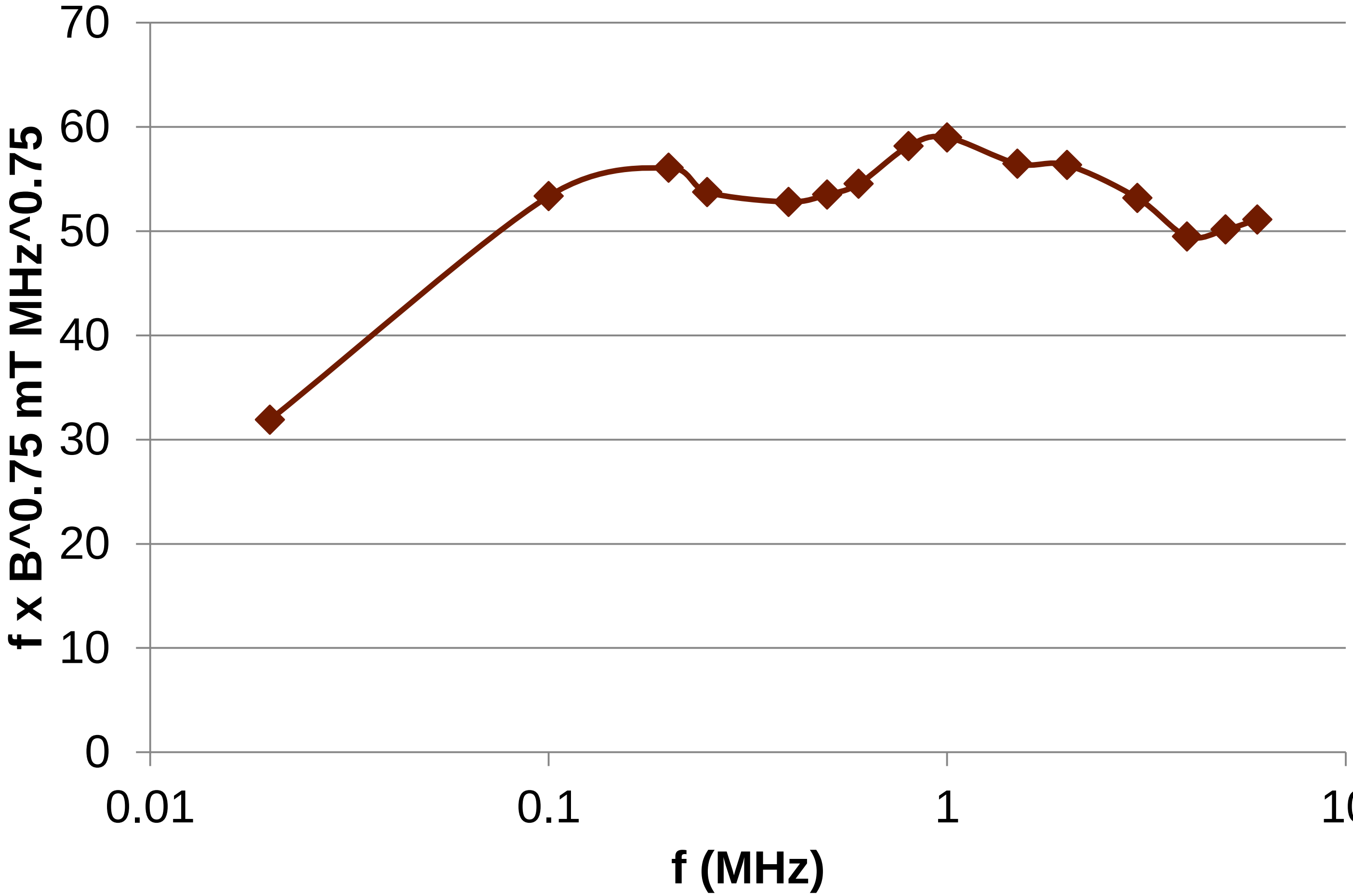
- Instead of fixing current, fix winding loss: $P_w = I^2 R_{ac}$
- Consider variation of R_{ac} with frequency.
  - Skin-effect limited $R_{ac} \propto f^{\,0.5}$
  - Current for constant P_w goes as $I \propto f^{\,-0.25}$
  - Modified performance factor PF = $Bf^{\,0.75}$
- Different assumptions about winding design lead to different exponents.

No significant ac resistance effects	PF = Bf
Fixed number of foil layers	PF = $Bf^{\,0.75}$
Fixed number of wire strands	PF = $Bf^{\,2/3}$
Fixed layer or strand thickness	PF = $Bf^{\,0.5}$

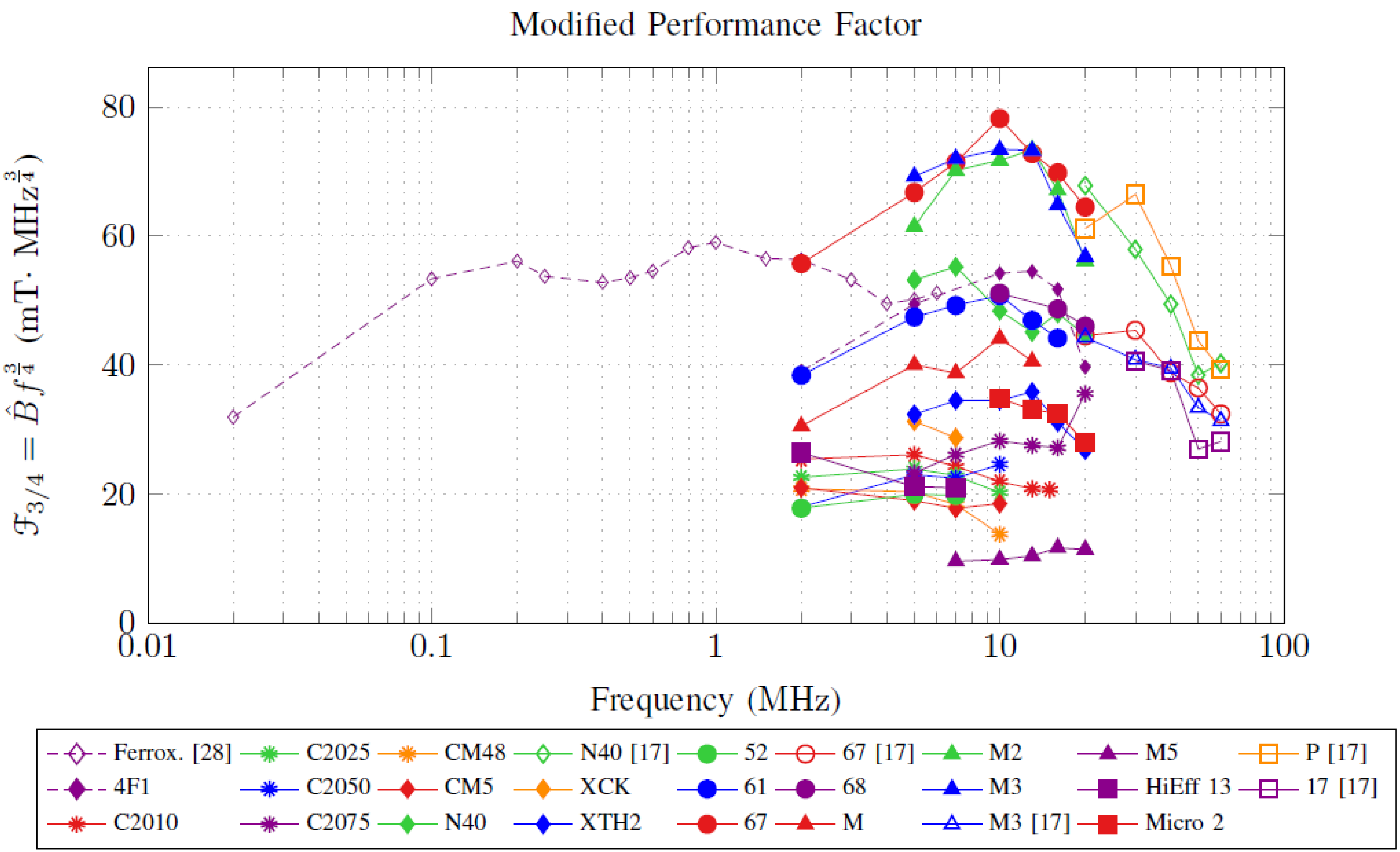
- Use PF = $Bf^{\,0.75}$ unless specific design scenario leads to a different choice.

Modified performance factor

- Based on considering skin effect in the winding
- $B{\cdot}f^{\,0.75}$

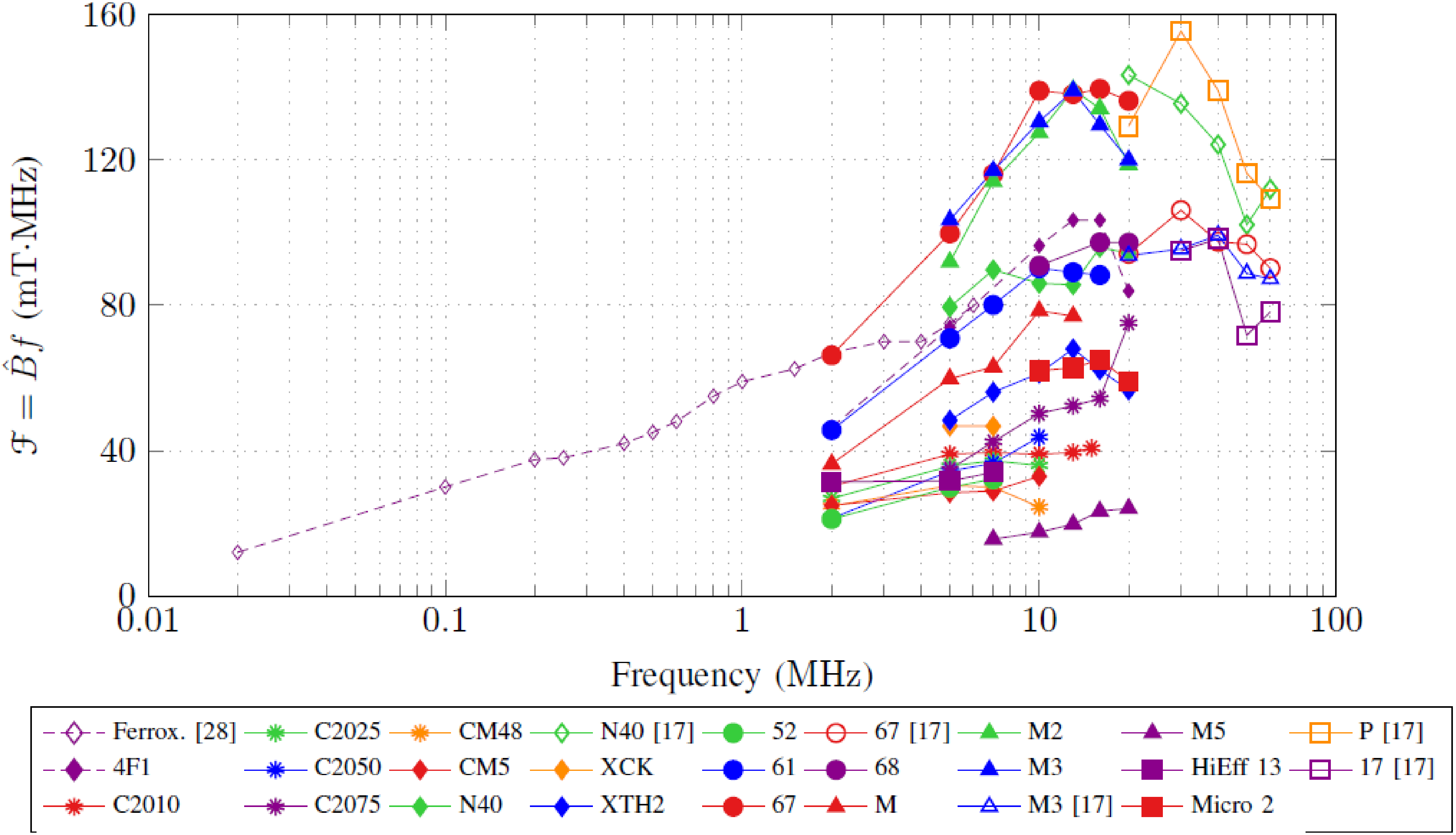


Adding data from more NiZn Ferrites



- Data from Perreault group at MIT;
  Hansen et al., *IEEE Trans. Pow. El.,* early access.

Standard performance factor



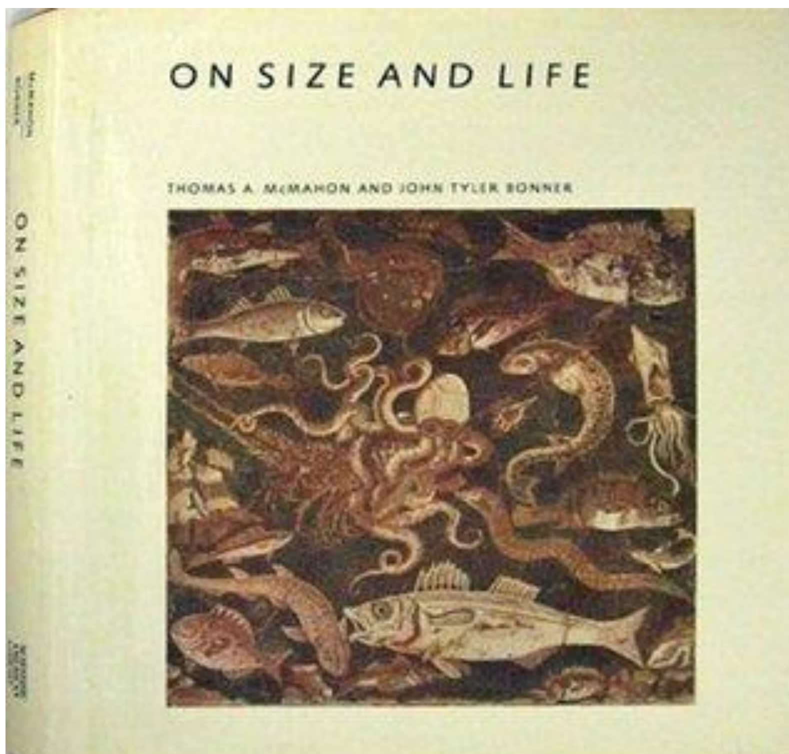
- 4X improvement vs. 100 kHz ... ***If you have a winding technology that makes proximity effect negligible.***

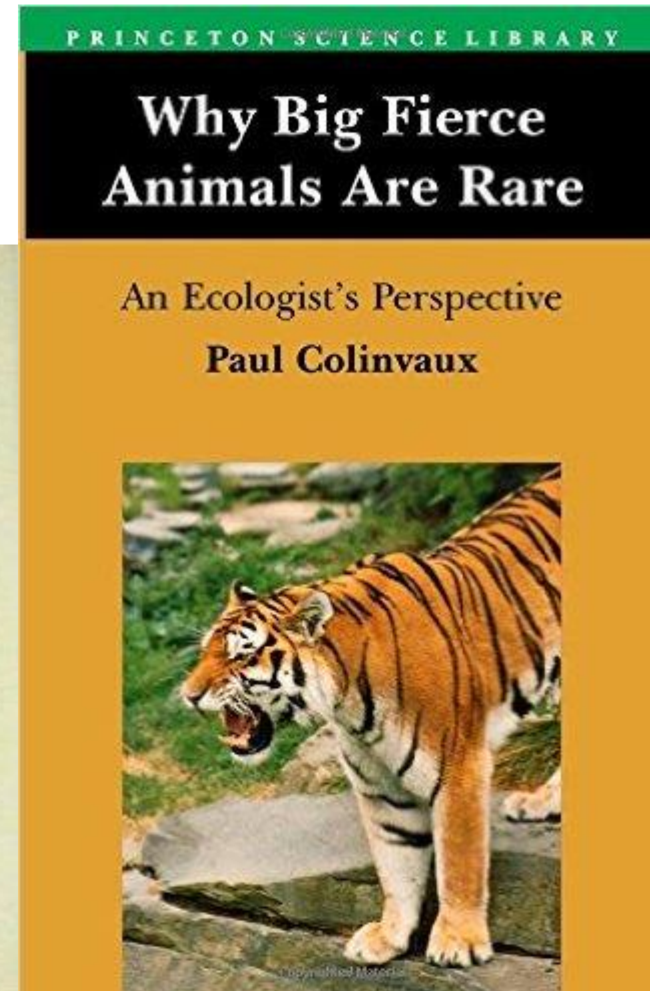
Conclusions

- Miniaturizing magnetics is fundamentally difficult.
- Scaling laws favor large components.
  - Use full size available.
  - Combine functions on one core when possible.
- Consider frequency increases.
  - Air-core is always better at higher frequency.
  - Magnetic-core frequency scaling depends on
    - Magnetic material performance: some good ferrites ~10 MHz.
    - High frequency winding capability
      - Need alternatives to litz in the MHz region.

Acknowledgements

- Title inspired by books:
  Why Big Fierce Animals are Rare by Paul Colinvaux
- *On size and Life* by Thomas A. McMahon and John T. Bonner.
- Research support from NSF, TI, and Enphase.

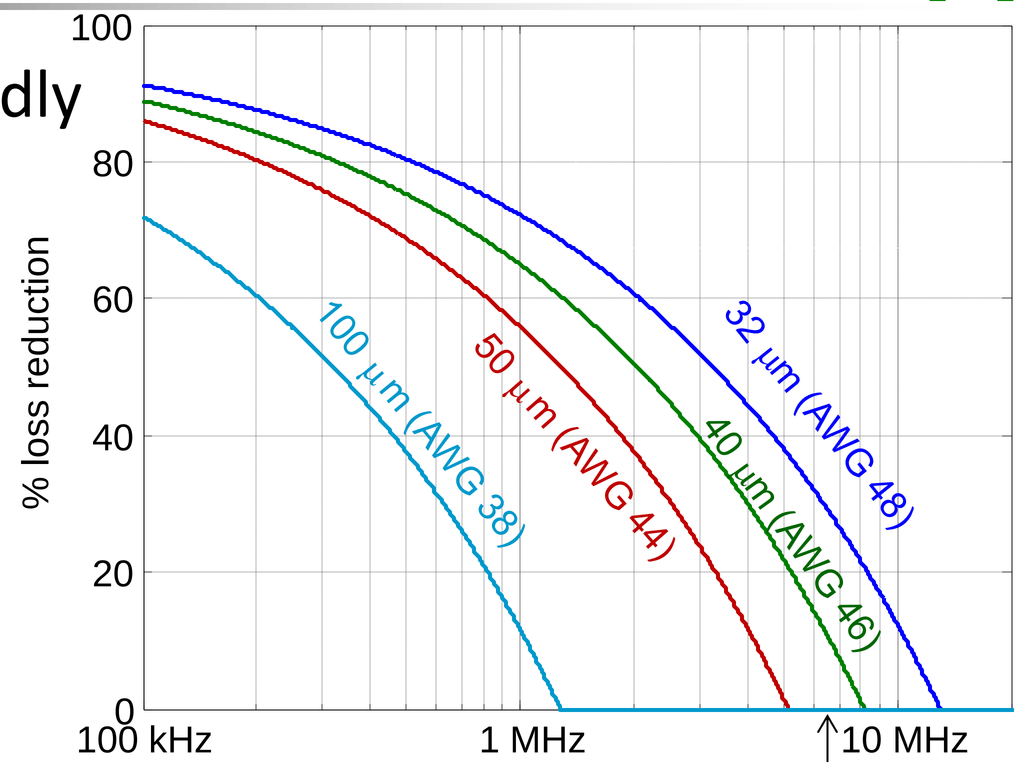




APPENDIX

Windings at MHz frequencies … Litz?

- Litz benefits drop off rapidly in the MHz range.
  - Barely better than a solid-wire winding.



- Huge room for improvement in theory:
  - A single-layer winding only has current in one skin depth: At 10 MHz, 21 μm.
  - 0.2% of a 1 cm winding window (0.23% with litz).
    → 400X improvement theoretically available.

Description of key references

Key references in high-frequency power magnetics with an emphasis on publications from our group and a focus on discrete components rather than chip-scale microfabricated components; for our perspective on the latter see [1].

For windings, Zimmanck's method can efficiently generate frequency dependent winding loss matrices for any geometry, 1D, 2D, or 3D, and use them to predict loss for different nonsinusoidal waveforms in any number of [2]. This method applies very generally, including to coupled inductors, wireless power transfer coils, etc. References cited in [2] provide more detailed background, including [26,27]. See also [28]. A systematic approach to generating full models for loss and simulation for 1D geometry is provided in [3]. To use 2D models effectively for 3D geometries such as E-cores, the strategy in [25] can reduce the error involved by a factor of 5.

Although the Dowell model is reasonably accurate, see the appendix of [9] for a simple correction that can enhance the accuracy. Also useful in the appendix of [9] is a simple effective frequency approach to address winding loss with non-sinusoidal windings.

Strategies to reduce proximity effect loss, using multiple thin layers or avoiding multiple layers, are compared in [6, 7, 8], considering different types of optimization constraints. An overview of the most common implementation of thin layers to reduce proximity effect loss, litz wire, is provided in [9]. A practical guide to using it is provided in [10], and the most complete model including effects of details of twisting construction, is in [11]. Approaches for using thin foil layers beyond frequencies where litz is practical are discussed in [12]. An implementation of these concepts for a resonant coil for applications such as wireless power transfer is described in [13]. For other applications, thin foil layers can have capacitance issues; circuits designs that reduce the voltage swing on the windings (e.g., [14]) can help reduce the impact of the capacitance.

The impacts of gap fringing and the quasi-distributed gap technique for reducing these problems are discussed in [15]. This reference includes data showing that a small gap is not effective for reducing the impact of fringing. With round-wire or litz-wire windings, shaping the winding can allow excellent performance with a standard gap [16].

In inductors with substantial dc resistance, two windings in parallel can be a good choice for good dc and ac resistance[17]. It is possible to extend this approach to applications in which the inductor carries a combination of line frequency ac current and high-frequency switching ripple, using, if needed, a capacitor to prevent low-frequency current from flowing through the high-frequency winding [18]. A foil winding with a semi-circular cutout region near the gap [19, 20, 21] can also be used to achieve a favorable ac/dc resistance combination.

Although copper windings are most common, aluminum can offer advantages if cost or weight are important [22, 23].

Performance factor for magnetic materials is described and extended in [24], and data on performance factor is provided for many materials in the MHz range. For core loss with non-sinusoidal waveforms, the iGSE model remains the standard method [4], although some of its limitations are now known, as discussed in [5].

References, p. 1 of 2

[1] C. R. Sullivan, D. Harburg, J. Qiu, C. G. Levey, and D. Yao, “Integrating magneticsfor on-chip power: A persepctive,” *IEEE Trans. on Pow. Electr.*, 2013.

[2] D. R. Zimmanck and C. R. Sullivan, “Efficient calculation of winding loss resistance matrices for magnetic components,” in *IEEE Workshop on Control and Modeling for Pow. Electr.*, 2010.

[3] M. Chen, M. Araghchini, K. K. Afridi, J. H. Lang, C. R. Sullivan, and D. J. Perreault, “A systematic approach to modeling impedances and current distribution in planar magnetics,” *IEEE Trans. on Pow. Electr.,*, 31(1), pp. 560–580, Jan 2016.

[4] K. Venkatachalam, C. R. Sullivan, T. Abdallah, and H. Tacca, “Accurate prediction of ferrite core loss with nonsinusoidal waveforms using only Steinmetz parameters,” in *IEEE Workshop on Computers in Pow. Electr.*, 2002.

[5] C. R. Sullivan, J. H. Harris, and E. Herbert, “Core loss predictions for general PWM waveforms from a simplified set of measured data,” in *IEEE Applied Power Electronics Conference and Exposition (APEC)*, Feb. 2010, pp. 1048–1055.

[6] M. E. Dale and C. R. Sullivan, “General comparison of power loss in single-layer and multi-layer windings,” in *IEEE Pow. Electr. Specialists’ Conf.*, 2005.

[7] M. E. Dale and C. R. Sullivan, “Comparison of single-layer and multi-layer windings with physical constraints or strong harmonics,” in *IEEE International Symposium on Industrial Electronics*, 2006.

[8] M. E. Dale and C. R. Sullivan, “Comparison of loss in single-layer and multi-layer windings with a dc component,” in *IEEE Ind. App. Soc. Ann. Mtg.*, 2006.

[9] C. R. Sullivan, “Optimal choice for number of strands in a litz-wire transformer winding,” *IEEE Trans. on Pow. Electr.*, vol. 14, no. 2, pp. 283–291, 1999.

[10] C. R. Sullivan and R. Y. Zhang, “Simplified design method for litz wire,” in *IEEE App. Pow. Electr. Conf. (APEC)*, 2014, pp. 2667–2674.

[11] C. R. Sullivan and R. Y. Zhang, “Analyticalmodel for effects of twisting on litz-wire losses,” in *IEEE Workshop on Control and Modeling for Pow. Electr. (COMPEL)*, 2014.

[12] C. R. Sullivan, “Layered foil as an alternative to litz wire: Multiple methods for equal current sharing among layers,” in *IEEE Workshop on Control and Modeling for Pow. Electr. (COMPEL)*, 2014.

[13] C. R. Sullivan and L. L. Beghou, “Design methodology for a high-Q self-resonant coil for medical and wireless-power applications,” in *IEEE Workshop on Control and Modeling for Pow. Electr. (COMPEL)*, 2013, pp. 1–8.

References, p. 2 of 2

[14] M. Chen, K. Afridi, S. Chakraborty, and D. Perreault. “A high-power-density wide-input-voltage-range isolated dc-dc converter having a multitrack architecture,” in *IEEE Energy Conversion Congress and Exposition (ECCE)*, 2015.

[15] J. Hu and C. R. Sullivan, “AC resistance of planar power inductors and the quasidistributed gap technique,” *IEEE Trans. on Pow. Electr.*, vol. 16, no. 4, pp. 558–567, 2001.

[16]] J. Hu and C. R. Sullivan, “Analytical method for generalization of numerically optimized inductor winding shapes,” in *IEEE Pow. Electr. Spec. Conf.*, 1999.

[17] A. van den Bossche and V. Valchev, *Inductors and Transformers for Power Electronics*. Taylor and Francis, 2005.

[18] C. Schaef and C. R. Sullivan, “Inductor design for low loss with complex waveforms,” in *IEEE App. Pow. Electr. Conf.*, 2012.

[19] J. D. Pollock and C. R. Sullivan, “Gapped-inductor foil windings with low ac and dc resistance,” in *IEEE Ind. App. Soc. Ann. Mtg.*, 2004, pp. 557–663.

[20] J. D. Pollock and C. R. Sullivan, “Modelling foil winding configurations with low ac and dc resistance,” in *IEEE Pow. Electr. Specialists’ Conf.*, 2005.

[21] W. Lundquist, V. Yang, and C. Castro, “Low ac resistance foil cut inductor,” in *IEEE Energy Conv. Cong. and Exp.*, 2014, pp. 2182–2186.

[22] C. R. Sullivan, “Aluminum windings and other strategies for high-frequency magnetics design in an era of high copper and energy costs,” *IEEE Trans. on Pow. Electr.*, vol. 23, no. 4, pp. 2044–2051, 2008.

[23] “Aluminum: The material of choice for transformers,” 2014, Siemens Industry, Inc.

[24] A. J. Hanson, C. R. Sullivan, and D. J. Perreault, “Measurements and performance factor comparisons of magnetic materials at MHz frequencies,” in *IEEE Energy Conv. Cong. and Exp.*, 2015; also early access in IEEE Trans. Pow. Electr.

[25] A. F. Hoke and C. R. Sullivan, "An Improved Two-Dimensional Numerical Modeling Method for E-Core Transformers", in *IEEE App. Pow. Electr. Conf.*, 2002.

[26] Xi Nan and C. R. Sullivan, “Simplified high-accuracy calculation of eddy-current loss in round-wire windings,” in *IEEE Pow. Electr. Spec. Conf*, 2004.

[27] C. R. Sullivan, “Computationally efficient winding loss calculation with multiple windings, arbitrary waveforms, and two- or three-dimensional field geometry,” *IEEE Trans. on Pow. Electr.*, vol. 16, no. 1, pp. 142–50, 2001.

[28] D. C. Meeker, “An improved continuum skin and proximity effect model for hexagonally packed wires,” *Journal of Computational and App. Mathematics*, vol. 236, no. 18, pp. 4635–4644, 2012.