Overview of Modelling Methods

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Winding models vs. Core models

- Linear, well known material properties.
- Behavior is a solution to Maxwell’s equations.
- Numerical, analytical, or mixed solutions.
- Can be accurately approximated by linear circuit networks, given enough RLC elements (usually just RL).

- Nonlinear material properties, known only through measurements.
- Models are behavioral, based on measurements.
  - Physics-based micromagnetic models exist, but can’t address ferrite loss yet.
- Circuit models based on RLC elements only can’t capture nonlinear behavior.
Winding models

Physical Design
Geometry & Materials

Electrical Measurements

Loss

Circuit model for simulation
Winding models

Current waveforms

Loss

Circuit model for simulation

Electrical Measurements

Physical Design
Geometry & Materials
Winding models

- Winding ac resistances?
Loss calculated from currents

- Conventional, incorrect, model for transformer winding loss (assume sine waves for now).
  - $P_{\text{winding}} = I_1^2 R_1 + I_2^2 R_2$
  - Problem: Loss varies drastically depending on relative phase/polarity.
  - Factor of 4 error in this case.
- Correct model options:
  - $R_1$ and $R_2$ that are only for specific phase relationship.
  - Resistance matrix.
Winding models

- Winding ac resistances?
- Frequency-dependent resistance matrix $R(f)$.
- Captures interactions between windings.
Winding models

See poster D09 on Thursday for how-to on this by Benedict Foo.
## Predictions from physical structure

<table>
<thead>
<tr>
<th></th>
<th>1-D fields</th>
<th>2-D or 3-D fields</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Rectangular conductors</strong> (e.g. foil and PCB)</td>
<td>Analytical</td>
<td>Numerical (Finite Element, PEEC, etc.)</td>
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<tr>
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<td><img src="image1" alt="Diagram" /></td>
<td><img src="image2" alt="Diagram" /></td>
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<tr>
<td><strong>Round-wire conductors</strong> (including litz)</td>
<td>Simulation-tuned physical model</td>
<td>Simulation-tuned physical model + dc field simulation</td>
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<td><img src="image3" alt="Diagram" /></td>
<td><img src="image4" alt="Diagram" /></td>
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Winding models: 1D, rectangular conductors

Physical Design
Geometry & Materials

Dowell, Spreen

M2SPICE (MIT)

Loss

Circuit model for simulation
Round conductor: Textbook problem

- Cylinder subjected to uniform field
- Dowell’s model is a crude approximation.
Textbook solution

- Exact solution, described by Bessel functions.
- Use for winding loss analysis pioneered by Ferreira.
Actual problem

- Array of cylinders subjected to uniform field
Using the Bessel solution for the real problem

Not a valid solution!

Real Solution (FEA)
Simulation Results

- Real behavior is between Dowell and Bessel.
- Sometimes closer to Dowell.
- Identical in low-frequency range with simple correction.
Xi Nan’s model

- Weighted average of Dowell-like and Bessel-like behavior: “Simulation tuned physical model”
- Fits experimental results better than Dowell or Bessel.
- Can be applied to 2D or 3D field configurations ...
Full winding loss model: 2-D, full frequency range, multi-winding interactions

- Hybridized Nan’s method (Zimmanck, 2010)
- Homogenization with complex permeability (Nan 2009, Meeker, 2012)

Available in FEMM
Winding models

Round wire/2D: “Hybridized Nan’s method”

Physical Design
Geometry & Materials

Current
waveforms

R(f)

M2SPICE
(MIT)

Electrical
Measurements

Loss

Circuit model
for simulation

?
Linear RL networks for winding models

- Three standard networks topologies that provide:
  - R increases with frequency.
  - L decreases with frequency.
- Can obtain identical behavior with any of the three.
- Can use any one to match measured behavior.

![Network Diagrams]

Foster
Cauer 1
Cauer 2
Core models

- Physics
- Flux waveforms
- Loss calculation
- Loss model
- Electrical Measurements
- Dynamic model
- Circuit simulation
Loss Calculation Models

- Steinmetz equation:
  - Sinusoidal waveforms only
- Various types of modified/generalized/etc. Steinmetz equations.
  - Extend to non-sinusoidal waveforms.
  - Most common: improved Generalized Steinmetz Equation (iGSE).
- Loss Map/Composite Waveform Method.
Loss Calculation Models

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Comments:

- iGSE vs. Loss Map:
  - Same predictions if you use the same data.
    - iGSE: sinusoidal data.
    - Loss Map
  - Loss map database can include dc bias effects.
  - iGSE can do any wave shape, whereas Loss Map is for rectangular only.
- Barg 2017 improves iGSE for extreme duty cycles.
- Weakness of most of these: “Dead time” affects loss in practice but not in the model. “Relaxation effects.”
Core simulation models

- Need to include nonlinearity.

- **Example:** Cauer 1 network to model saturation behavior and frequency-dependent permeability in nanocrystalline tape-wound cores.

  - Successfully matched pulse behavior in high-amplitude operation (Sullivan and Muetze, IAS 2007)
  - Did not examine loss behavior.

- Open question: what model structures capture dynamic nonlinear behavior correctly?
Conclusions

- **Winding loss:**
  - Complex but feasible to model accurately.
  - For 2 or more windings, need resistance *matrix*.
  - 1D rectangular conductors: analytical solutions.
  - 2D rectangular conductors: numerical simulations.
  - 1D or 2D round wire: Simulation-tuned physical models are better than Dowell or Bessel.

- **Core loss**
  - Nonlinear and can only be found experimentally.
  - Open questions on data needed and models.
References


Spreen
Zimmanck
Meeker
Nan
Sullivan and Muetze, IAS 2007
Appendix:
Further slides for reference or questions
Relaxation effect

Assumption: *Energy* loss per cycle doesn’t change if waveform pauses. Loss only when dB/dt > 0.

Voltage

Flux

Cumulative Energy Loss

Assumed *without* physical basis

(Core loss reference 7, 13)
Measurements prove assumption wrong (Core loss reference 7, 13)

- Increase in loss per cycle with increasing off-time (ferrite (below)).
- Not observed in powdered iron (right).

![Graph showing loss vs. time for different voltages and cycle times.](image-url)
iGSE (improved Generalized SE)

- Based on $P(t) = k_i (\Delta B)^w \left\lvert \frac{dB}{dt} \right\rvert^z$, plus compatibility with Steinmetz equation for sine waves.
- Result: $P(t) = k_i (\Delta B)^{\beta-\alpha} \left\lvert \frac{dB}{dt} \right\rvert^\alpha$
- Formula to get $k_i$ from sinsoidal data:
  $$k_i \approx \frac{k}{2^{\beta+1} \pi^{\alpha-1} \left( 0.2761 + \frac{1.7061}{\alpha + 1.354} \right)}$$
- Formula for PWL waveforms:
  $$P_v = \frac{k_i (\Delta B)^{\beta-\alpha}}{T} \sum_m \left\lvert \frac{B_{m+1} - B_m}{t_{m+1} - t_m} \right\rvert^\alpha (t_{m+1} - t_m)$$
Rectangular conductors with 1-D fields

- Dowell’s analysis is correct, but limited.
  - Parallel windings and layers?
  - Extract inductance matrix as well as resistance matrix?
- M2Spice: automatic generation of a SPICE model for a 1D system of windings.
  - Minjie Chen (Princeton) and Dave Perreault (MIT)
- Result:
Litz wire

- Basic model is the same as for round wire.
- Full model now available to predict effect of construction.
- Simple rules to avoid problems due to poor construction can avoid the need for the full model.
1D, 2D and 3D modeling approaches

- 1D: can use analytical models.
  - For Xfrmers and good (quasi-) distributed gap Ls.
    - Dowell isn’t precise but we know how to do better.
- 2D: Fast, easy, low-cost simulations.
  - Naïve sections for E-cores can be misleading.
  - Mimic return path for to reduce error 5X.
- 3D: Use for verification, not design.