Lumped model to explain and approximate dimensional effects in ferrite cores

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Core size affects magnetic material performance

- A larger core has:
  - Higher losses in power applications.
  - Lower frequency rolloff in EMC applications.
- Two effects cause this:
  - Skin effect resulting from finite resistivity of ferrite.
  - “Dimensional resonance” effects: EM wave propagation is slow because of high $\varepsilon_r$ and $\mu_r$ so interior can be out of sync with exterior.

Four sizes of 3C95 MnZn ferrite toroids

(Marcin Kacki, SMA magnetics)
Examples of theoretical behavior: plots of $|B|$ in a round centerpost

- Skin effect, affected by $\mu$ and $\sigma$ (permeability and conductivity)

- Wave propagation, affected by $\mu$ (permeability) and $\varepsilon$ (permittivity = dielectric const.)

How to predict these effects

- Need inherent permeability, permittivity, and conductivity of the material.
  - Permittivity and conductivity are not on typical datasheets.
  - Strong function of frequency.
  - Different for different materials.

- Can solve Maxwell’s equations if we assume linearity.
  - 2D FEA: most solvers don’t include both E and B in one solution.
  - 3D FEA is expensive in engineering time, computation time, and software licensing.
  - Analytical: possible for some cases.
Proposed alternative

- A lumped, ladder network approximation to the distributed behavior of the real system.
- This has been done before to model eddy currents in cores.
  - Also similar to standard models in many other applications, e.g. transmission line models.
- Now we include E field and displacement behavior as well.

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Why this approach?

Why not a standard numerical solution to the partial differential equation, FEA or otherwise, in 1D or 2D?

- This is another option; not necessarily better.
- The model is accessible to electrical engineers:
  - The model structure can help provide intuition.
  - It can be solved by standard circuit simulators.
- The model can be incorporated into a simulation of a higher-order system, with or without model order reduction.
- Natural routes to extend to a nonlinear model.

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An intuition pump

- $\varepsilon_r$ can be on the order of 100,000 ($10^5$)

- An air capacitor with 50 mm diameter plates, 1 mm apart, has ~ 17.5 pF.

- With $\varepsilon_r = 10^5$, a 5 mm diameter rod, 100 mm long, metalized at the ends has 175 pF.
Consider concentric shells of a toroidal core

Broken up conceptually, but it's still solid material
Basis of lumped model

• Each path (blue dashed line) has capacitance and resistance, in parallel:
  • \( R = \frac{\ell \rho}{A} \)
  • \( C = \frac{\varepsilon_r \varepsilon_0 A}{\ell} \)

where \( \ell \) is the length of that loop, and \( A \) is its cross section, \( A \approx (2\pi r)\Delta x \).

• Each path links a little less flux, as we move from the outer shells to the inner shells.

• The inductance arising from a shell divides half and half: half outside the blue line (not linked) and half inside (linked)

• Inductance of a shell
  \[ \Delta L = \frac{N^2 A}{(2\pi r)} = \frac{(\Delta x \ell)}{(2\pi r)} \]
  (for simplicity assume \( N = 1 \))
Full lumped model

- Model for one shell

- Transmission line model for multiple shells

- This is a linear model, but any and all of the components can be made nonlinear and accurately capture nonlinear aspects of the corresponding behavior.

- Add an ideal transformer at the input for $N > 1$.

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Can the lumped model predict this effect?

- We will use measured data for $\mu^*(f)$, $\varepsilon^*(f)$ (complex permeability and complex permittivity).
- Complex permeability is derived from measurement of small toroid.
- Complex permittivity is derived from measurements of disk samples.

Four sizes of 3C95 MnZn ferrite toroids

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Good match to key features

Analysis and figure: Marcin Kacki, SMA magnetics
My results

Four sizes of 3C95 MnZn ferrite toroids

- T6/3/4
- T50/30/16.5
- T87/56/20
- T152/104/24

Normalized Impedance vs. Frequency (MHz)
Individual effects

Conductive and displacement currents both matter.
Individual effects

Conductive and displacement currents both matter.
Variation of intrinsic properties w/ frequency

Complex Permeability

Conductivity

Real part of permittivity

Relative permeability

Real part of relative permittivity

imaginary

real
Is this model useful?

- Requires data on complex permeability and permittivity vs. frequency.
- These parameters also vary as a function of temperature.
- Doesn’t yet include nonlinearity or hysteresis—but these can be included by using nonlinear elements in the ladder.
- About 15 “rungs” in the ladder network is sufficient.
- Model as structured here works only at one frequency. For a wideband simulation model, replace each branch with a ladder network—the model becomes a ladder of ladders.
- Model order reduction techniques could match the behavior for faster simulations.
Data requirements: for each temperature:

- Complex permeability and complex permittivity of small samples over frequency: Two measurements.
- Alternative: One measurement of effective complex permeability for each core size.
- Conclusion: fewer measurements needed to address a wide range of core sizes.
Visualize flux density vs. radius

Graph showing relative flux density vs. radius at 50 kHz and 300 kHz.
Visualize flux density vs. radius
Visualize flux density vs. radius
Visualize flux density vs. radius
Solutions

- Breaking up core into small sticks or sheets can avoid these problems.
  - Same concept as steel laminations for line frequency, but addresses displacement current as well as conduction current.
  - Avoids the need for the complex model as well as reducing losses and extending frequency range of a filter.

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References on similar models without displacement current

- Basic model topology has been used before for conduction eddy currents, e.g. for laminations in these references: