able for further investigation of possible flux migration phenomena.

5   Steinmetz Curve Fits

The Steinmetz[4] approximation,

$$\overline{P_v} = k f^\alpha \hat{B}^\beta, \tag{7}$$

where $\overline{P_v}$ is the average power density, f is the excitation frequency, and $\hat{B}$ is the peak flux density, is commonly used to characterize core loss data for sinusoidal excitation, but can also be applied to our square-wave data. We first describe a formulation in terms of frequency and flux density, but then also provide a formulation in terms of pulse widths and applied voltage.

To linearize the equation for curve fitting, we used base-10 logarithms (referenced to $1\,\mathrm{V}$ or $1\,\mathrm{s}$), because the preferred values for voltage and time used in the experiment are round decilog values, and because using $\log_{10}$ makes it convenient to express the standard error in familiar units of decibels (i.e., $10\log_{10}(P/P_{\mathrm{ref}})$). This choice does not affect the values of the k, α, and β parameters.

In a typical case, Ferroxcube 3C81 material, fitting (7) to the entire set of square wave data gave a standard error of about $1.5\,\mathrm{dB}$. Visually inspecting the plots shows a distinct increase in slope around $100\,\mathrm{kHz}$. This inspired a six-parameter, two-plane curve fit,

$$\overline{P_v} = \max(k_1 f^{\alpha_1} \hat{B}^{\beta_1} + k_2 f^{\alpha_2} \hat{B}^{\beta_2}) \tag{8}$$

This fits the data to two intersecting planes that function like a single plane with a fold in it (Figure 11). It fits the 3C81 data with a standard error of $0.35\,\mathrm{dB}$, much better than the single equation, and also better than simply using different parameters for different frequency ranges.

Note that while the formula we are fitting is essentially the same form as the Steinmetz equation, and we use the variable names k, α, and β, this is a *different* model, because is is based on square wave experimental data, and is intended to predict rectangular pulse core losses. In situations where this might cause confusion, we will subscript the present parameters[5] to distinguish them from the classic Steinmetz, sinusoidal, parameters.

The boundary between the two planes (the fold) projected onto the $\log_{10}(f)$-$\log_{10}(\hat{B})$ plane is a straight line,

$$\log_{10}(\hat{B}) = a_0 + a_1 \log_{10}(f) \tag{9}$$

where

$$a_0 \quad = \quad \frac{\log_{10}(k_1/k_2)}{\beta_2 - \beta_1}$$

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[4]Named for C. P. Steinmetz's work [7], although Steinmetz did not include the frequency dependence that is now standard [8].

[5]We choose 'r', for "rectangular," thus avoiding 's' which might suggest either "square" or "sinusoidal."
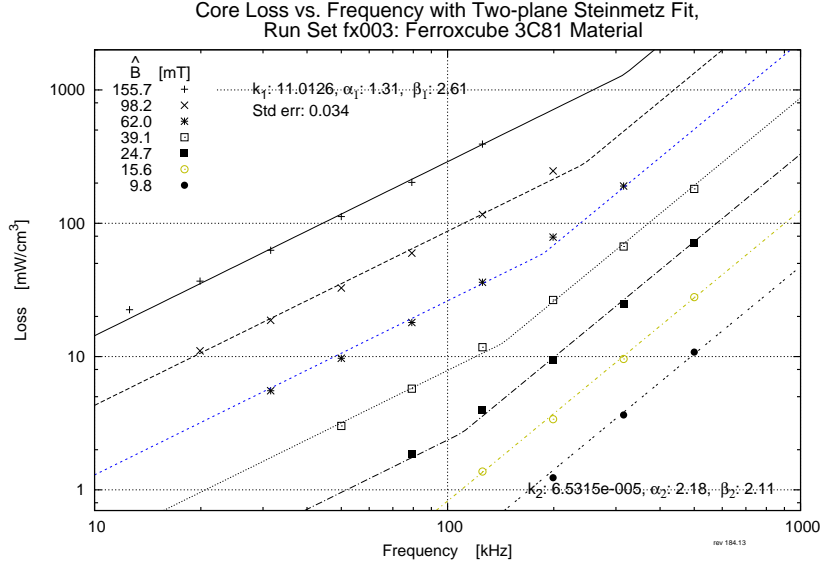
Figure 11: Two-plane Steinmetz curve fit to Ferroxcube 3C81 material, square wave loss data.

$$a_1 = \frac{\alpha_1 - \alpha_2}{\beta_2 - \beta_1}.$$

The parameters a_0 and a_1 might provide a useful characterization of the transition of the dominating loss mechanisms from hysteresis to eddy currents, for comparing different materials. The two-plane Steinmetz parameters from our experiments are listed in Table 2.

This method of curve fitting has advantages over the common practice of providing independent Steinmetz parameters for different frequency ranges:

1. There is no step discontinuity at frequency range boundaries.

2. There is no arbitrary choice of boundary frequency. The curve-fitting optimization chooses the best boundary line (i.e., choosing a_0 and a_1).

3. The curve fitting optimization uses *all* the data.

There is no significant computational penalty for using this model, and coding is easy; programmers can use either (8) directly, or (7), using k_1, α_1, and β_1 for $\log_{10}(\hat{B}) > a_0 + a_1 \log_{10}(f)$, and $k_2, \alpha_2,$ and β_2 otherwise.

The two-plane Steinmetz parameters from our experiments are listed in Table 2. The table includes two different sets of k values. The lowercase k_1 and k_2 are the values used in (8), implicitly using reference values of 1 T and 1 Hz. Also listed are uppercase K_1 and K_2 based on reference values $f_0 = 100\,\text{kHz}$

Manufacturer Material	Geom	Set	k_1	K_1	α_1	β_1	γ_1	k_2	K_2	α_2	β_2	γ_2
Ceramic Magnetics Inc												
MN60	T	cm01	6.085	86810	1.32	2.47	2.15	899.8μ	67879	2.00	2.13	1.13
MN8CX	T	cm02	63.01	182100	1.19	2.49	2.3	177.4μ	87357	2.20	2.29	1.09
Ferroxcube												
3C81	T	fx003	11.01	91200	1.31	2.61	2.3	65.32μ	41432	2.18	2.11	0.93
3C81	E	fx09	18.02	88800	1.23	2.45	2.22	350.0μ	52769	2.10	2.33	1.23
3C90	T	fx010	36.86	39570	1.19	2.94	2.75	2.895μ	18223	2.39	2.16	0.77
3F3	T	fx004	102.4	67200	1.13	2.81	2.68	11.93μ	28414	2.30	2.14	0.84
3F3	E	fx05	40.63	65950	1.14	2.50	2.36	224.8μ	38221	2.12	2.36	1.24
Magnetics Inc.												
F	T	mi005	26.41	72920	1.24	2.76	2.52	7.612μ	32369	2.37	2.22	0.85
K	T	mi007	246.2	86830	1.10	2.95	2.85	5.276μ	20750	2.41	2.48	1.07
L	T	mi08	706.8	150800	1.04	2.87	2.83	276.1m	99927	1.69	2.88	1.19
P	T	mi003	10.91	43090	1.28	2.80	2.52	75.99μ	36099	2.16	2.13	0.97
R	T	mi01-6	30.16	67220	1.25	2.90	2.65	14.55μ	30033	2.31	2.24	0.93
W	T	mi02	832.7m	131000	1.51	2.37	1.86	10.59m	123857	1.82	2.04	1.22

Table 2: Two-plane Steinmetz parameters for various magnetic materials in two general geometries, toroidal (T) and E-core (E). The k_i parameters are referenced to $f = 1\,\mathrm{Hz}$ and $\hat{B} = 1\,\mathrm{T}$; the K_i parameters are referenced to $f = 100\,\mathrm{kHz}$ and $\hat{B} = 100\,\mathrm{mT}$. All the k_i and K_i parameter have dimensions of $\mathrm{W/m}^3$. The α and β parameters for use in (8); γ is required for (11).

and $\hat{B}_0 = 100\,\mathrm{mT}$, to be used in

$$\overline{P_v} = \max\left(K_1(f/f_0)^{\alpha_1}(\hat{B}/\hat{B}_0)^{\beta_1} + K_2(f/f_0)^{\alpha_2}(\hat{B}/\hat{B}_0)^{\beta_2}\right) \qquad (10)$$

Both are included because the (8) is simpler to use, but the values of K_1 and K_2 are more physically meaningful, because they are based on results near the range of values used in practice, rather than on values many orders of magnitude different.

For example, the values of K for one core shape are very similar to those for another core shape and the same material, indicating that the losses predicted by the model are very similar for the two shapes. The values of k, on the other hand, can be very different, but this is only an artifact of the use of the 1 T and 1 Hz reference points, which are distant from the actual operating point, and is not an indication that the predictions in the region of interest are significantly different.

The results tend to indicate slightly smaller losses for E cores than for toroidal cores made with the same material. This does not actually indicate of superior performance for E cores. Rather, it is a result the effective area and effective core length provided one the core datasheet, which we used in our calculations.

5.1 Model in Terms of Pulse Width and Voltage

The model (8) can be reformulated in terms of pulse width and voltage. For direct use with that composite with form hypothesis, we also reformulated it to represent the energy loss for one pulse, rather than the average power loss over a whole cycle. This results in energy loss per pulse, per unit volume, of

$$E_{v,a} = \max\left(\frac{k_1}{(NA)^{\beta_1}2^{(\beta_1+\alpha_1)}}V_a^{\beta_1}t_a^{\gamma_1} \ , \ \frac{k_2}{(NA)^{\beta_2}2^{(\beta_2+\alpha_2)}}V_a^{\beta_2}t_a^{\gamma_2}\right) \qquad (11)$$

where V_a is the voltage applied during a pulse of duration t_a, A is the cross-section area of the core, and N is the number of turns. The parameter γ is provided in Table 2, for convenience, but can also be simply calculated from α and β:

$$\gamma = 1 + \beta - \alpha. \qquad (12)$$

The application of this formula is illustrated in Appendix B.

6 Dead-Time Loss

In Phase I, we discovered a significant deviation from the CWH during periods of prolonged constant flux. This was most evident with the `expand` waveform, in which a square wave is stretched by inserting off-time between the square pulses—thus increasing the period while holding the pulse widths constant. For example, in Figure 12 we plot loss versus off time, t_0. The "predicted" value is simply the loss measured for the characterization square-wave measurement. In contrast, the CWH works well for the asymmetric waveform (Figure 13), which has no dead time

The effect is also noticeable in plots of the `skew` waveform (Figure 14). The core loss is uniformly higher than for square waves, at about the same value as seen for the `expand` waveform having the same period. Unlike the `expand` waveform, the plot is not significantly slopped—the increased loss due to greater off time in one part of the waveform is about offset by the decreased loss due to lesser off time in the other part.

In Phase II, we want to determine if the phenomenon is real, and if so, possibly characterize it.

6.1 Measurement Artifacts

In order to check for possible errors introduced by the apparatus, we repeated a representative experiment with (1) a DC current probe, and (2) increased bridge blocking capacitance. Neither precaution made a significant difference.

The magnetization current is routinely measure using a Tektronix P6021 AC current probe (set to $10\,\mathrm{mA/mV}$). For this experiment we substituted a Tektronix TCP303 current probe. As an extra precaution, we did the probe degaussing procedure before each run. (Run set `mi05-5`)

B Example Application

This section will demonstrate an example of using the two-plane Steinmetz fit parameters in Table 2 to calculate loss for an unusual waveform. The loss will be calculated using the conventional Steinmetz parameters of frequency and flux amplitude, and then the example will be repeated using voltage and pulse width as parameters. The design parameters for the example are shown in Table 4, and the waveform in Figure 16. The calculation of core loss would be identical whether this were an inductor or a transformer, and so we do not specify which. In the case of a transformer, there might be different numbers of turns and voltages on different windings, but the number of turns and corresponding voltage on any one winding would be adequate information.

B.1 Conventional Parameters

For the calculation based on frequency in flux density, we first calculate the amplitude of the flux density,

$$B_{\text{peak-to-peak}} = \frac{\int v dt}{NA} = \frac{5 \cdot 75 \text{ V}\mu\text{s}}{20 \cdot 154.8 \times 10^{-6} \text{ m}^2} = 0.12112 \text{ T} \qquad (20)$$

$$\hat{B} = \frac{B_{peak-to-peak}}{2} = 0.06056 \text{ T} \qquad (21)$$

Next, we need the square-wave loss for that flux amplitude, and for each of the pulse widths in the waveform: 5 μs and 7.5 μs. A square wave with a pulse width of 5 μs would have a period of 10 μs, and thus a frequency of 100 kHz. A square wave with a pulse width of 7.5 μs would have a period of 15 μs, and a frequency of 66.7 kHz. Based on the parameters in Table 2 for 3C90 material, we can calculate the square wave loss from (8). To do that, we calculate first with k_1, α_1, and β_1, and then again with k_2, α_2, and β_2, and we select the larger of the two calculated values. For example, at 100 kHz, for k_1, α_1, and β_1,

$$P_{v,1} = 36.86 \cdot (100 \times 10^3)^{1.19} \cdot (0.061)^{2.94} = 8819 \text{ W/m}^3 \qquad (22)$$

The results for these calculations for each of the two frequencies and for each of the two parameter sets are shown in Table 5.

Parameter	Value
Core shape	PQ32/30
Core material	Ferroxcube 3C90
Effective core area, A	154.8 mm^2
Effective core volume	10.44 cm^3
Number of turns, N	20

Table 4: Example design parameters.
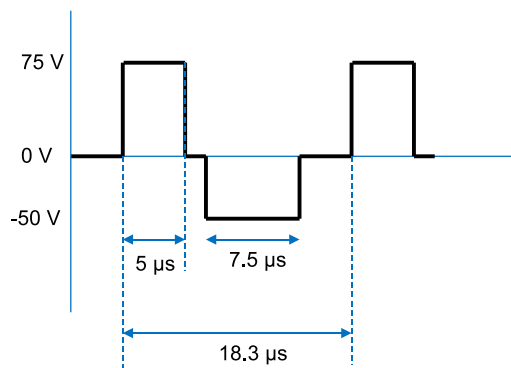
Figure 16: Example design voltage waveform applied across the winding of the device.

Plane	100 kHz	66.7 kHz
k_1, α_1, β_1	8.63 kW/m^3	5.33 kW/m^3
k_2, α_2, β_2	6.04 kW/m^3	2.29 kW/m^3

Table 5: Calculated square-wave loss.

From Table 5, we choose the maximum value in each column, to implement the maximum function in (8). In this case, the values in the first row of the table are the maxima for both frequencies.

We now have the square-wave power loss values, and in preparation for using the composite waveform hypothesis, we convert the form of these values to obtain the energy lost in each pulse. For the positive, 5 μs pulse, the energy loss is simply the pulse width multiplied by the corresponding power per unit volume: $E_a = 5\ \mu\text{s} \cdot 8.63\ \text{kW/m}^3 = 43.2\ \text{mJ/m}^3$ per pulse. Similarly, for the negative pulse, $E_b = 7.5\ \mu\text{s} \cdot 5.33\ \text{kW/m}^3 = 40.0\ \text{mJ/m}^3$. The total loss for cycle is the sum of the loss for each of these pulses, or 83.2 mJ/m^3. We can now convert that back to a power loss number by dividing by the total period, 18.3 μs, which results in a power per unit volume $P_v = 4.54\ \text{kW/m}^3$. Based on the fact that kW/m^3 are the same as mW/cm^3, we can multiply that loss number by the volume in cm^3 to obtain the final power loss in milliwatts:

$$P = 4.64 \cdot 10.44 = 47.4 \text{ mW} \tag{23}$$

One can easily automate this calculation using Excel, Matlab, or any programming language, but it has been presented here in terms of the manual process in order to ensure that all the steps are clear without reference to any particular programming language.

B.2 Pulse width and Voltage as Parameters

In this case, we directly apply (11), repeated here for convenience:

$$E_{v,a} = \max \left(\frac{k_1}{(NA)^{\beta_1} 2^{(\beta_1 + \alpha_1)}} V_a^{\beta_1} t_a^{\gamma_1} \ , \ \frac{k_2}{(NA)^{\beta_2} 2^{(\beta_2 + \alpha_2)}} V_a^{\beta_2} t_a^{\gamma_2} \right) \qquad (24)$$

We substitute in values from Table 4 and values of parameters $k_1 = 36.86$, $\alpha_1 = 1.19$, $beta_1 = 2.94$ and $\gamma_1 = 2.75$ for the first set and $k_2 = 2.895$, $\alpha_2 = 2.39$, $beta_2 = 2.16$ and $\gamma_2 = 2.77$ for the second. We obtain, for 5 μs and 75 V, $E_{v,a} = 43.2$ mJ/m^3, and, for 7.5 μs and 50 V, $E_{v,b} = 40.0$ mJ/m^3, exactly the same as was found with the other approach. Any difference would only occur as a result of roundoff errors tracking through the calculation.

As before, the total loss for cycle is the sum of the loss for each of these pulses, or 83.2 mJ/m^3. We can now convert that back to a power loss number by dividing by the total period, 18.3 μs, which results in a power per unit volume $P_v = 4.54$ kW/m^3, and an average power loss of 47.4 mW.