# Composite Waveform Hypothesis

Edward Herbert Revised December 7, 2014

The Pilot Project core loss study sponsored by PSMA at Dartmouth analyzed the *composite waveform hypothesis* and determined that: "Despite the minor discrepancies, the loss prediction method yields higher accuracy, and is easier to use, than other methods for non-sinusoidal waveforms."

The "minor discrepancies" largely are attributable to loss of energy in the core during off-time between pulses when the duty-ratio is less than unity (the off-time phenomenon).

## *PSMA sponsored studies at Dartmouth*

The Pilot Project began in the Spring of 2009. Data was taken for one ferrite core and one powdered metal core. It was observed that increasing the off-time between excitation pulses increased the core loss per cycle in the ferrite core.[1]

The Phase II Project began in the Spring of 2010. The Phase II project tested the composite waveform hypothesis on a variety of cores of different materials, with emphasis on ensuring that the off-time core loss phenomenon was not just a test rig or test procedure artifact.[2]

The Phase II study confirmed that the off-time phenomenon is seen on a variety of cores of different shape and material, and it provided a large amount of data for further analysis.

The off-time core loss phenomenon was verified independently by Jonas Mühlethaler.[3]

## *Herbert curves*

The Herbert curve is an alternative way of displaying core loss data in a form that is directly useable for calculating core loss for low duty-ratio rectangular excitation using the composite waveform hypothesis. For more accurate loss predictions for a specific component, core loss data can be taken on that component, which avoids the complexity and errors of trying to convert bulk material data to various core shapes and winding configurations.

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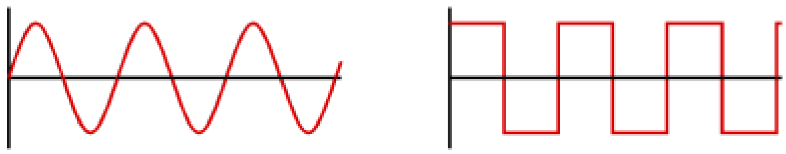
[1] **Testing Core Loss for Rectangular Waveforms**, February 7, 2010 by Charles R. Sullivan and John H. Harris, Thayer School of Engineering at Dartmouth and Edward Herbert. The Pilot Project data can be downloaded from **http://www.psma.com/coreloss/pilot**

[2] **Testing Core Loss for Rectangular Waveforms, Phase II Final Report**, 21 September 2011 by Charles R. Sullivan and John H.Harris; Thayer School of Engineering at Dartmouth. The Pilot Project data can be downloaded from **http://www.psma.com/coreloss/phase2**

[3] **Improved Core Loss Calculation for Magnetic Components Employed in Power Electronic Systems**, Muhlethaler, J.; Biela , J.; Kolar , J. W.; Ecklebe , A.; Power Electronic Systems Laboratory, Zurich, Switzerland; IEEE Transactions on Power Electronics, Vol. 27, No. 2, February 2012.

## \Use rectangular excitation

Core loss data traditionally are taken using sine wave excitation. Most power converters use rectangular wave excitation, often at reduced duty-ratio. It is recommended that square-wave excitation be used for core loss testing.



Sine wave data would make more sense if it were possible to apply Fourier analysis, but Dr. Sullivan showed that core losses of non-sinusoidal waveforms cannot be accurately predicted by separately examining the Fourier components of the waveform.[4]

## Use electrical units

Core loss traditionally is calculated using magnetic units and frequency. Magnetic units are unfamiliar to circuit design engineers, and converting between magnetic units and electrical units is a source of confusion and error. To graph core loss as a function of flux density $\hat{B}$, the flux density $\hat{B}$ must be calculated. Graphing as a function of the excitation voltage V and pulse-width t requires no conversion–the points are just plotted.

## Avoid area, volume, density and turns

Core loss tables traditionally give the results in power density (W/m$^3$).

If the focus is the core loss of a particular component, the geometric calculations can be avoided by measuring the loss of that specific component. In the Herbert curves, the x-axis is the pulse-width of the square-wave excitation in time µs, the graphs are curves of constant applied excitation voltage V, and the loss is read directly as power in watts W on the y-axis.

This avoids converting to and from magnetic units, and avoids all of the geometric density calculations, each a possible source of error. By taking data using a specific component, all of its dimensions and physical idiosyncrasies are accounted for in the data, so arcane parameters such as "effective core area" and "effective core volume" are avoided.

If the data is for an un-wound core, the voltage may be expressed as volts/turn.

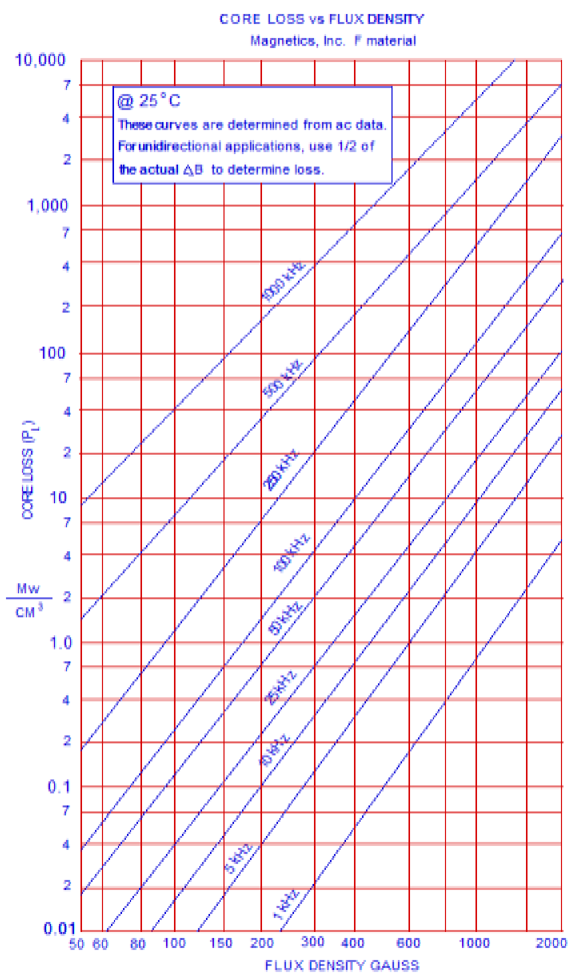
If the core loss graph is for a material, length, area, flux density and power density units cannot be avoided. The voltage is expressed as volts per turn meter$^2$, and the power is watts per meter$^3$.

[4] **Testing Core Loss for Rectangular Waveforms, Phase II Final Report**, 21 September 2011 by Charles R. Sullivan and John H.Harris; Thayer School of Engineering at Dartmouth

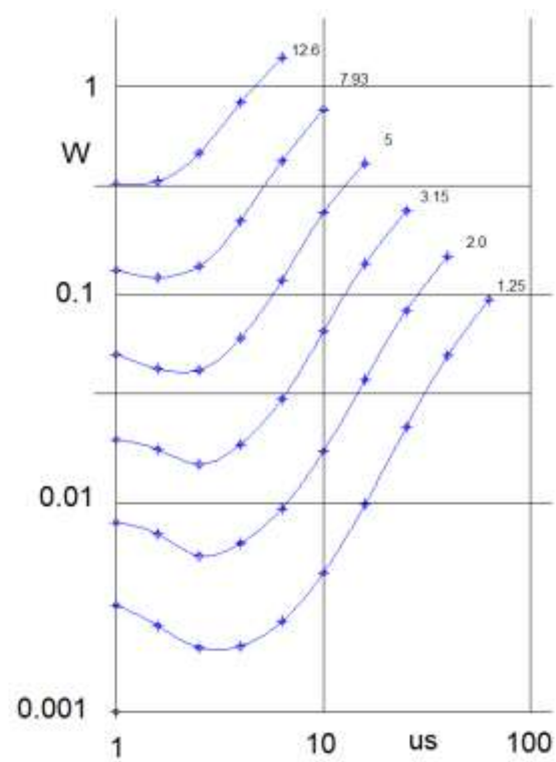
## *Core loss Graphs*

A representative traditional core loss graph is shown on the left below. The loss is given as mw/cm$^3$, requiring determination of the volume and multiplying. The x-axis is in flux density $\hat{B}$, requiring a fairly complicated calculation of voltage, frequency and cross-sectional area.

By contrast, a representative Herbert core loss graph for a component is shown on the right. The data are taken using square-wave excitation, with the x-axis being the pulse-width and the various curves being the applied voltage with a 5-turn winding. The core loss is read directly as Watts.




The graph below uses square-wave data from Phase III, for the runs mi12-2-001 through mi12-2-045.
Magnetics Inc. F-42206 toroid core with five turns.




Traditional core loss graph:

Power density vs flux density $\hat{B}$ for various frequency $f$ curves.

The Herbert core loss graph:

Power vs pulse width for various applied square-wave excitation voltages.

The test run are the same for either style of graph: Each point is a test. Excitation of a certain waveform, frequency and voltage is applied to the core, and its loss is measured.

## *Correlation of the traditional curves and the Herbert curves*

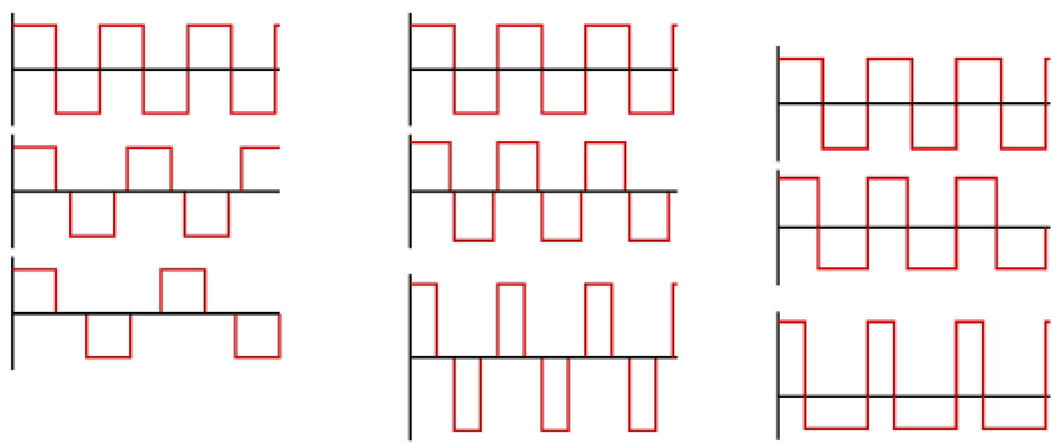
Exactly the same tests are run to generate either the traditional core loss curves or the Herbert curves. However, if the goal is to design pulse-width-modulated (pwm) power converters with low duty-ratio rectangular pulses, it is preferred to start with square-waves.

In theory, a traditional graph could be converted to a Herbert graph, and *vice-versa*. The problem arises in writing the formulae with sufficient accuracy so that the conversion factors can be applied. The familiar Steinmetz equation can be converted, but it is straight line approximation (on a log-log graph) of a curved function, and the errors can be large.

Some properties of the core loss are much easier to visualize on the Herbert curves. For a given core with a given winding and excitation voltage, it is well known that the losses decrease as the pulse-width decreases (frequency increases). That is easily seen in the Herbert curve, but almost impossible to visualize in the traditional curve. It also can be seen very easily that the core loss decreases with frequency only to a point, then it reverses and the losses increase.

## *Low duty-ratio rectangular excitation*

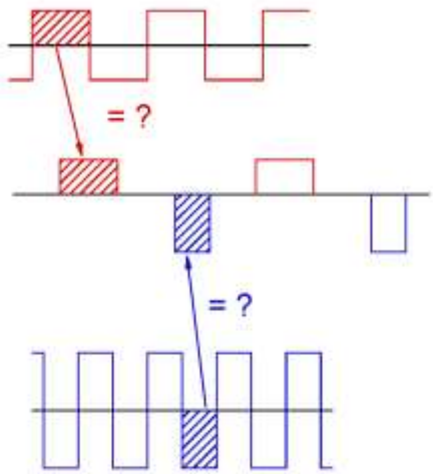
Many power converters use low duty-ratio rectangular wave excitation. While there are many possibilities, three waveforms are particularly important, "expanded" waveform, "buck" waveform, and "asymmetrical" waveform, as shown below. In the expanded waveform, the pulse-width and voltage are constant, but the period increases as off-time is inserted between the pulses. In the buck curves, the period and the flux in volt-seconds are constant. As the pulse-width is decreased (increasing off-time), the voltage is increased such that the voltage * time = a constant. The assymetrical wave-form, shown on the right above, has no off-time, but the voltage of the excitation differs for the positive and negative pulses. In all cases, the flux must be equal at steady state conditions, or flux walking will occur. This means that the product of the voltage and time is equal for the positive and negative portions of the cycle.



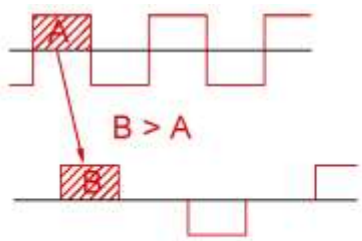
Extensive data were taken for expanded excitation in the Dartmouth Core Loss studies. Unfortunately, no core loss data was taken for buck curves. Data were taken for asymmetric excitation, but its validity is doubtful, as a blocking capacitor was used.

## ***Composite waveform hypothesis.***

The premise of the *composite waveform hypothesis* is that the core loss can be determined by the voltage and pulse-width. An isolated pulse, as in a low duty-ratio waveform, is hypothesized to have the same loss as a pulse of the same amplitude and pulse-width in a square-wave. If that were true, the losses of low duty-ratio waveforms could be estimated from square-wave data.




That proved not to be the case.

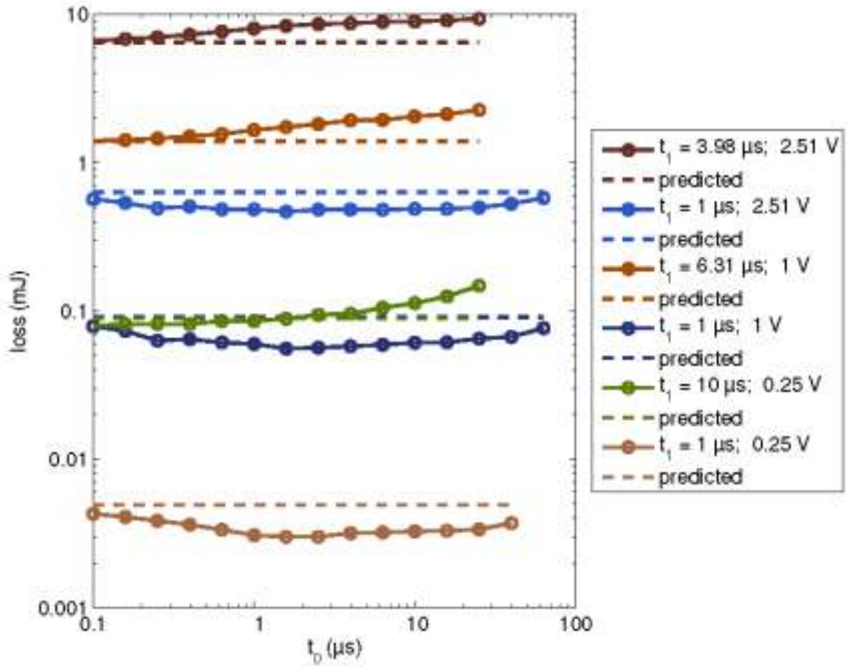



Adding off-time between the pulses increased the loss per pulse significantly.

## Ferrite core

The off-time effect was more pronounced in the ferrite core. The effect was quite strong, with energy increases as much as 30 percent.

A lot of work was done taking extensive data and ensuring to the extent possible that the effect was not due to test rig anomalies.




## ***Core loss calculations using the composite waveform hypothesis***

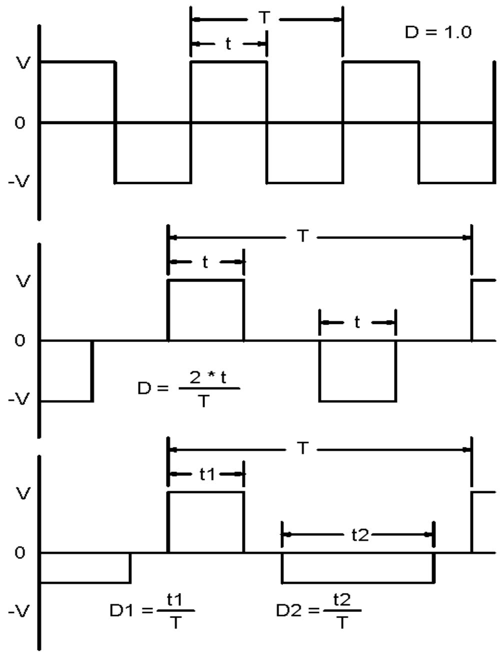
The figures below define pulse-width and duty-ratio: In all cases, the pulses are repetitive steady-state pulses, as would be generated in a pwm converter at stead-state conditions.

**For a square-wave excitation**, *t* is the pulse-width and *T* is the period. The duty-ratio *D* is 1.0. The core loss is read directly on the Y axis for the corresponding excitation voltage curve, though interpolation is necessary if the excitation voltage falls between the curves. (Use caution, interpolating log curves is tricky.).

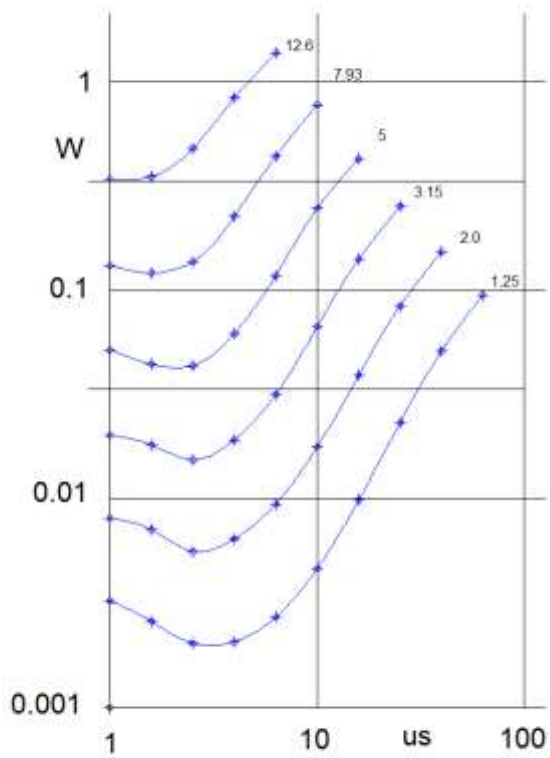
For expanded pulsed excitation, *t* is the pulse-width and *T* is the period. The duty-ratio *D* is 2 * *t / T*. To estimate the core loss, read the core loss on the Y axis for the corresponding excitation voltage curve and pulse width *t*, then multiply the result by the duty-ratio *D*.

For an asymmetrical pulsed excitation, the volt-seconds must be equal for the positive and negative pulses. T is the period, *t1* is the positive pulse-width, *t2* is the negative pulse-width, *V1* is the excitation voltage of the first pulse, and *V2* is the excitation voltage of the second pulse. Two duty-ratios are defined, *D1 = t1 / T* and *D2 = t2 / T*.

To estimate the core loss for the asymmetrical waveform, first estimate the power of the positive pulse by reading the power on the Y axis for the corresponding its excitation voltage *V1* and pulse width *t1*, then multiply the result by its duty-ratio *D1*. Next, estimate the power of the negative pulse by reading the power on the Y axis for the corresponding its excitation voltage *V2* and its pulse width *t2*, then multiply the result by its duty-ratio *D2*. Add the partial results to get the total estimated core loss.




Times and duty-ratios defined.




The Herbert curve shown above is for a Magnetics Inc. F-42206 toroid core with five turns. The curves are the square-wave excitation voltage at the terminals, so the volts/turn would be 1/5$^{th}$. The x-axis is the pulse width t, NOT the period. Power is read directly on the Y axis.