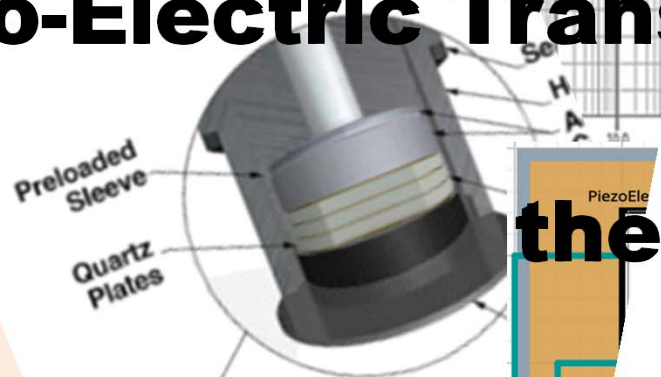


An Introduction to Piezo-Electric Transducer Models: the P-E Pressure Sensor



Norman Elias

Why, What, How

Simulations support Component Selection, Signal Conditioner Design, Controls, Worst Case and Statistical Design

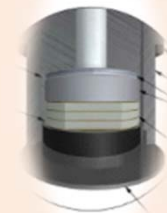


Modeling objectives:

Parameterized → Reusable

Datasheet-Based → Easy to use

Complete, Reliable, Accessible

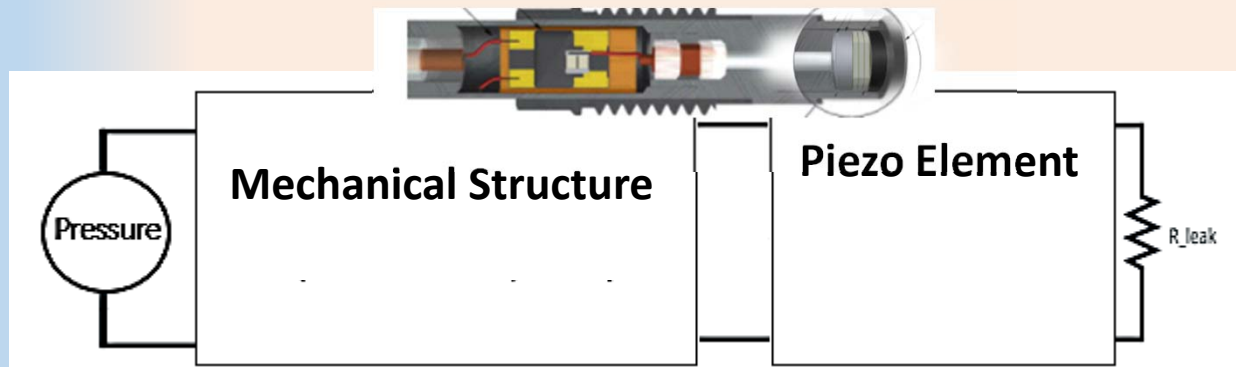


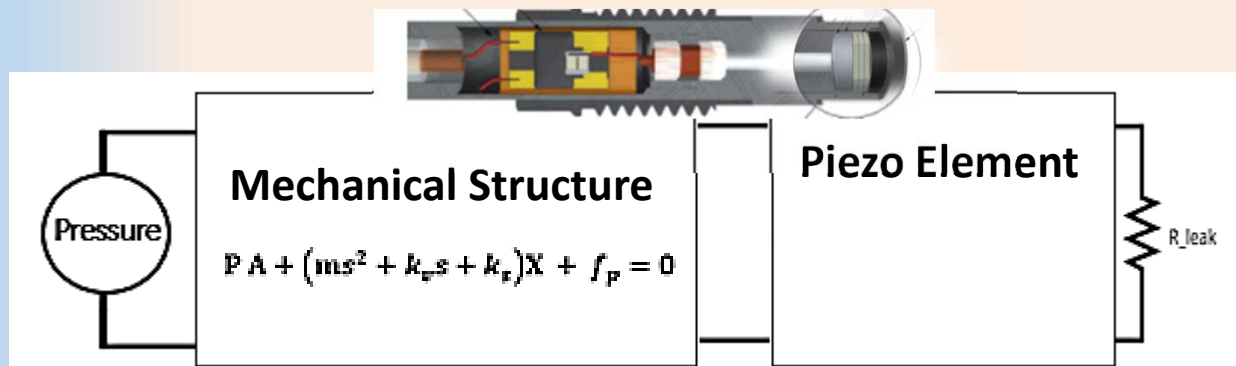
A piezoelectric pressure sensor model

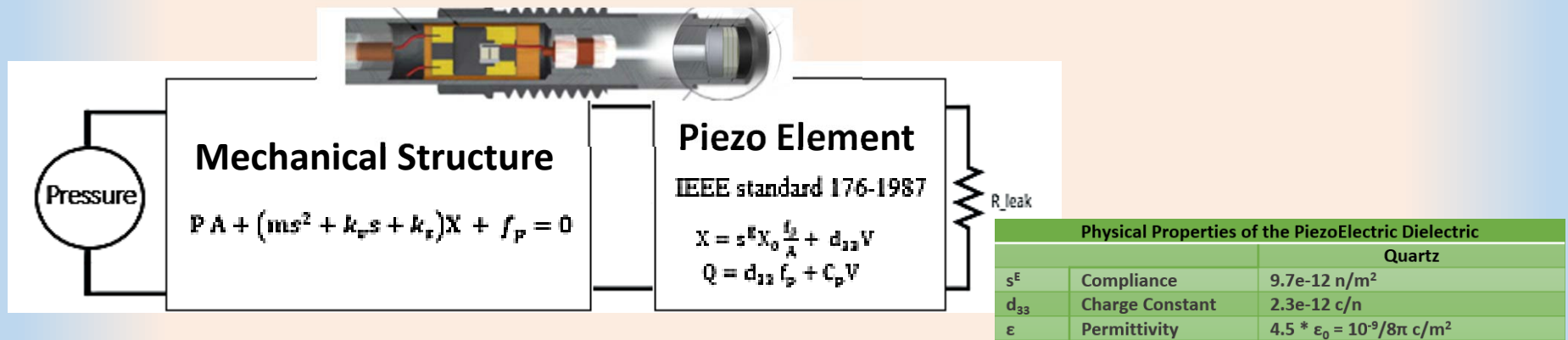
Equations, Implementation, Some observations,

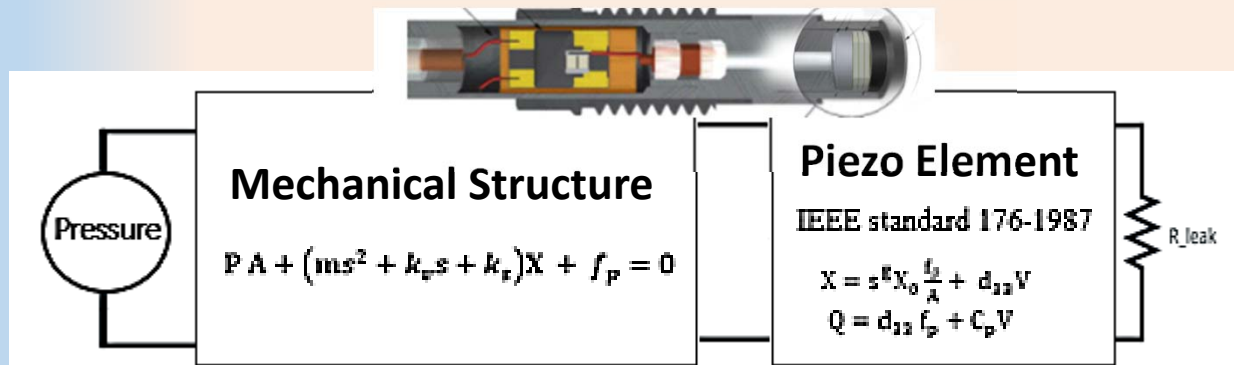
A (free) copy for you to try







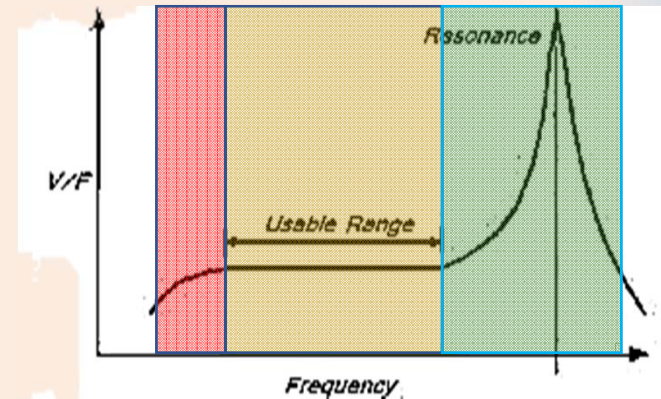
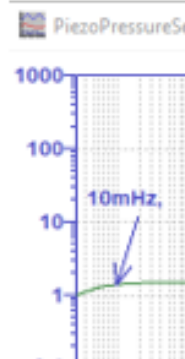




MODEL: 113B26

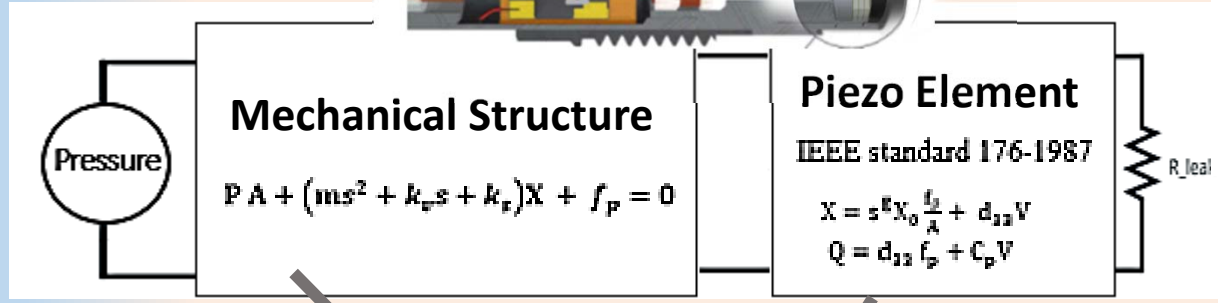
Platinum Stock Products; High frequency ICP[®] pressure sensor, 500 psi, 10 mV/psi, 0.218" dia. diaphragm accel. comp.

- Measurement Range: (for ±5V output) 500 psi(3450 kPa)
- Sensitivity: (±10%) 10 mV/psi(1.45 mV/kPa)
- Low Frequency Response: (-5%) 0.01 Hz
- Resonant Frequency: >=500 kHz(>=500 kHz)
- Electrical Connector: 10-32 Coaxial Jack
- Weight: (with clamp nut) 0.20 oz(6.0 gm)



| Physical Properties of the PiezoElectric Dielectric | | |
|---|-----------------|---|
| Quartz | | |
| s^E | Compliance | $9.7 \cdot 10^{-12} \text{ n/m}^2$ |
| d_{33} | Charge Constant | $2.3 \cdot 10^{-12} \text{ c/n}$ |
| ϵ | Permittivity | $4.5 * \epsilon_0 = 10^{-9} / 8\pi \text{ c/m}^2$ |





MODEL: 113B26
 Platinum Stock Products; High frequency ICP® pressure sensor, 500 psi, 10 mV/psi, 0.218" dia. diaphragm accel. comp.

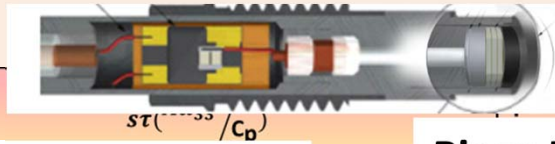
- Measurement Range: (for ±5V output) 500 psi(3450 kPa)
- Sensitivity: (±10%) 10 mV/psi(1.45 mV/kPa)
- Low Frequency Response: (-5%) 0.01 Hz
- Resonant Frequency: >=500 kHz(>=500 kHz)
- Electrical Connector: 10-32 Coaxial Jack

$$k_s \ll \frac{C_p}{\epsilon s^E}$$

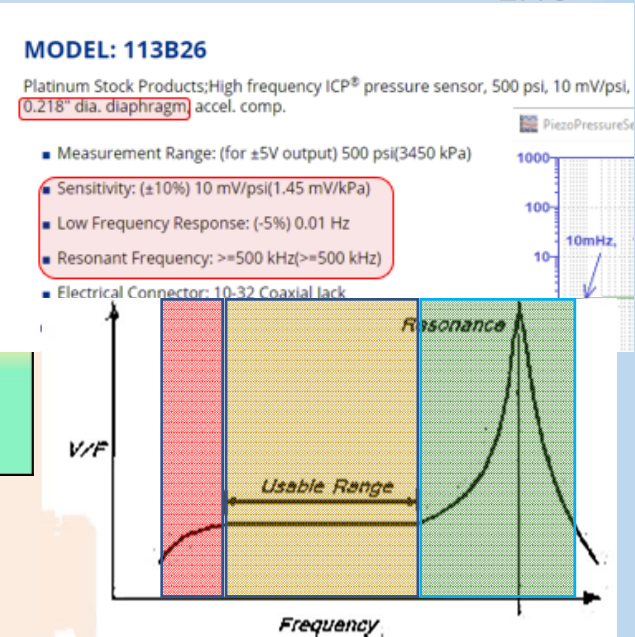
$$\frac{V}{P} = \frac{s A d_{33} \tau}{(\epsilon s^E m s^2 + \epsilon s^E k_v s + C_p) \left(1 + s \tau \left(1 - \frac{d_{33}^2}{\epsilon s^E} \right) \right) + C_p \left(\frac{d_{33}^2}{\epsilon s^E} \right) s \tau}$$

| Physical Properties of the PiezoElectric Dielectric | | |
|---|-----------------|--|
| Quartz | | |
| s^E | Compliance | 9.7e-12 n/m ² |
| d_{33} | Charge Constant | 2.3e-12 c/n |
| ϵ | Permittivity | 4.5 * $\epsilon_0 = 10^{-9}/8\pi$ c/m ² |





| | | |
|-----------------------------|--|--|
| Low Frequencies to Mid-Band | $s\tau \left(\frac{Ad_{33}}{C_p} \right)$ | 1-linearity |
| Mechanical Structure | | |
| High Frequency | $PA + (ms^2 + k_p s + k_r)X + f_p = 0$ $\left(\frac{\epsilon s^E - d_{33}^2}{C_p} \right) ms^2 + \left(\frac{\epsilon s^E - d_{33}^2}{C_p} \right) k_v s +$ | Piezo Element IEEE standard 176-1987 $X = s^E X_0 \frac{d_{33}}{A} + d_{33} V$ $Q = d_{33} f_p + C_p V$ |



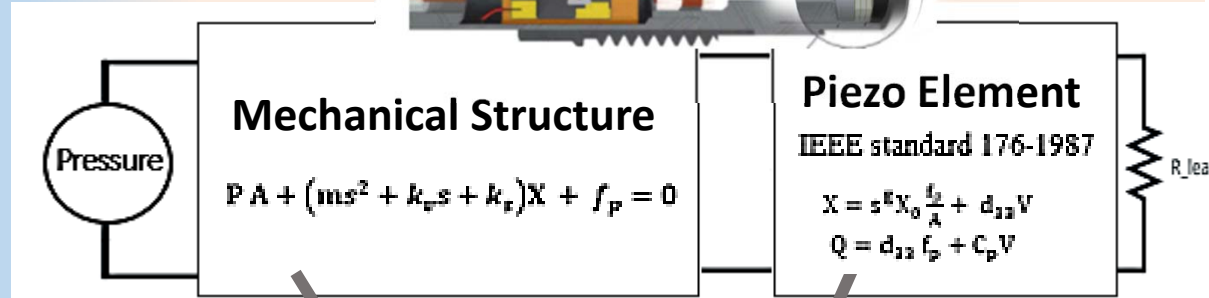
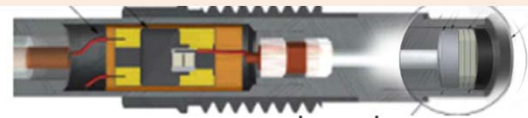
$$k_s \ll \frac{C_p}{\epsilon s^E}$$

$$\frac{V}{P} = \frac{sAd_{33} \tau}{\left(\epsilon s^E ms^2 + \epsilon s^E k_v s + C_p \right) \left(1 + s\tau \left(1 - \frac{d_{33}^2}{\epsilon s^E} \right) \right) + C_p \left(\frac{d_{33}^2}{\epsilon s^E} \right) s\tau}$$

| | | |
|-----------------------------|---|--|
| Low Frequencies to Mid-Band | $\frac{s\tau \left(\frac{Ad_{33}}{C_p} \right)}{(1 + s\tau)}$ | 1-linearity $\tau = \frac{1}{\omega_{min} \sqrt{\text{linearity}(2\text{-linearity})}}$ $C_p = \frac{Ad_{33}}{\text{sensitivity}}$ |
| High Frequency | $\frac{A d_{33} / C_p}{\left(\frac{\epsilon s^E - d_{33}^2}{C_p} \right) ms^2 + \frac{\left(\epsilon s^E - d_{33}^2 \right) k_v s + 1}{C_p}}$ | $m = \frac{C_p}{\omega_0^2 (\epsilon s^E - d_{33}^2)}$ $k_v = \frac{2\zeta C_p}{\omega_0 (\epsilon s^E - d_{33}^2)}$ |

Physical Properties of the PiezoElectric Dielectric

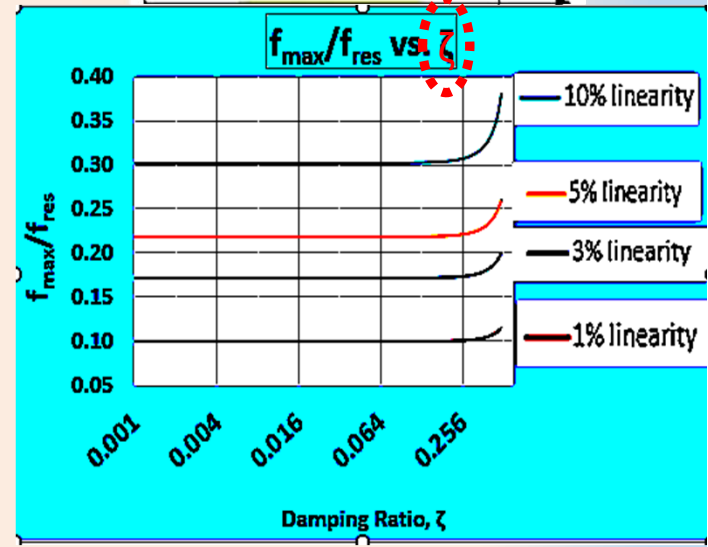
| Quartz | | |
|------------|-----------------|--|
| s^E | Compliance | 9.7e-12 n/m ² |
| d_{33} | Charge Constant | 2.3e-12 c/n |
| ϵ | Permittivity | 4.5 * $\epsilon_0 = 10^{-9}/8\pi$ c/m ² |



MODEL: 113B26
 Platinum Stock Products; High frequency ICP® pressure sensor, 500 psi, 10 mV/psi, 0.218" dia. diaphragm accel. comp.

- Measurement Range: (for ±5V output) 500 psi(3450 kPa)
- Sensitivity: (±10%) 10 mV/psi(1.45 mV/kPa)
- Low Frequency Response: (-5%) 0.01 Hz
- Resonant Frequency: >=500 kHz(>=500 kHz)
- Electrical Connector: 10-32 Coaxial Jack

$$\frac{V}{P} = \frac{K}{(\epsilon s^E E_{ms}^2 + \epsilon s^E (s/\omega_0)^2 + 2\zeta(s/\omega_0) + 1)}$$



| | | |
|-------------------------|--|--|
| Frequencies to Mid-Band | $\frac{sT(Ad_{33}/c_p)}{(1+sT)}$ | $T = \frac{1\text{-linearity}}{\omega_{min}/\text{linearity}(2\text{-linearity})}$ |
| Frequency | $\frac{A d_{33}/c_p}{\left(\frac{\epsilon s^E - d_{33}^2}{c_p}\right) ms^2 + \frac{\epsilon s^E - d_{33}^2}{c}}$ | $C_p = Ad_{33}/\text{sensitivity}$ |
| | | $m = \frac{C_p}{\omega_0^2(\epsilon s^E - d_{33}^2)}$ |
| | | $k_p = \frac{2\zeta C_p}{\omega_0(\epsilon s^E - d_{33}^2)}$ |

Physical Properties of the PiezoElectric Dielectric

| | | Quartz |
|-----------------|-----------------|--|
| s ^E | Compliance | 9.7e-12 n/m ² |
| d ₃₃ | Charge Constant | 2.3e-12 c/n |
| ε | Permittivity | 4.5 * ε ₀ = 10 ⁻⁹ /8π c/m ² |

From Equations

$$P A + (ms^2 + k_v s + k_s)X + f_p = 0$$
$$X = s^2 X_0 \frac{1}{A} + d_{32} V$$
$$Q = d_{32} f_p + C_p V$$

$$T = \frac{1\text{-linearity}}{\omega_{\min} \sqrt{\text{linearity}(2\text{-linearity})}}$$
$$C_p = \frac{A d_{33}}{\text{sensitivity}}$$

$$m = \frac{C_p}{\omega_0^2 (\epsilon \epsilon^E - d_{33}^2)}$$
$$k_v = \frac{2\zeta C_p}{\omega_0 (\epsilon \epsilon^E - d_{33}^2)}$$

to Simulations

VHDL-AMS on Siemens PartQuest™ Explore

via

Equivalent Circuit on LTspice



From Equations

$$P A + (ms^2 + k_v s + k_s)X + f_p = 0$$

$$X = s^2 X_0 \frac{A}{k_s} + d_{33} V$$

$$Q = d_{33} f_p + C_p V$$

$$T = \frac{1\text{-linearity}}{\omega_{\min} \sqrt{\text{linearity}(2\text{-linearity})}}$$

$$C_p = A d_{33} / \text{sensitivity}$$

$$m = \frac{C_p}{\omega_0^2 (e s^2 - d_{33}^2)}$$

$$k_v = \frac{2 \zeta C_p}{\omega_0 (e s^2 - d_{33}^2)}$$

to Simulations

VHDL-AMS on Siemens PartQuest™ Explore

via

Equivalent Circuit on LTspice

Use voltage, charge and current to represent mechanical force, Displacement and velocity

| | Mechanical | Electrical |
|---------|------------------------------------|------------------------|
| Across | Force | Voltage |
| Through | Velocity | Current |
| Mass | $F = m \, d(\text{velocity}) / dt$ | $V = m \, di/dt$ |
| Damping | $F = k_v \cdot \text{velocity}$ | $V = k_v \cdot i$ |
| Spring | $\text{displacement} = F/k_s$ | $i = (1/k_s) \, dv/dt$ |

From Equations

$$P A + (m s^2 + k_v s + k_s) X + f_p = 0$$

$$X = s^2 X_0 \frac{A}{A} + d_{32} V$$

$$Q = d_{32} f_p + C_p V$$

$$T = \frac{1\text{-linearity}}{\omega_{\min} \sqrt{\text{linearity}(2\text{-linearity})}}$$

$$C_p = \frac{A d_{33}}{\text{sensitivity}}$$

$$m = \frac{C_p}{\omega_0^2 (\epsilon s^E - d_{33}^2)}$$

$$k_v = \frac{2\zeta C_p}{\omega_0 (\epsilon s^E - d_{33}^2)}$$

to Simulations

VHDL-AMS on Siemens PartQuest™ Explore

via

Equivalent Circuit on LTSpice
Use voltage, charge and current to represent mechanical force, Displacement and velocity

| | Mechanical | Electrical |
|---------|------------------------------------|------------------------|
| Across | Force | Voltage |
| Through | Velocity | Current |
| Mass | $F = m \, d(\text{velocity}) / dt$ | $V = m \, di/dt$ |
| Damping | $F = k_v \cdot \text{velocity}$ | $V = k_v \cdot i$ |
| Spring | Displacement $\neq F/k_s$ | $i = (1/k_s) \, dv/dt$ |

```

.params kappa_sq=(d33/epsilon)
.params Cp=Area*d33/sensitivity
.params mval=1/((epsilon*kappa_sq-
.params Kv=2*zeta*w_0*kappa_sq
.params E_V2F=0-(Cp/epsilon)

```

From Equations

$$P A + (m s^2 + k_v s + k_s) X + f_p = 0$$

$$X = s^2 X_0 \frac{A}{\omega} + d_{33} V$$

$$Q = d_{33} f_p + C_p V$$

$$T = \frac{1\text{-linearity}}{\omega_{\min} \sqrt{\text{linearity}(2\text{-linearity})}}$$

$$C_p = A d_{33} / \text{sensitivity}$$

$$m = \frac{C_p}{\omega_0^2 (\epsilon s^E - d_{33}^2)}$$

$$k_v = \frac{2\zeta C_p}{\omega_0 (\epsilon s^E - d_{33}^2)}$$

to Simulations

via

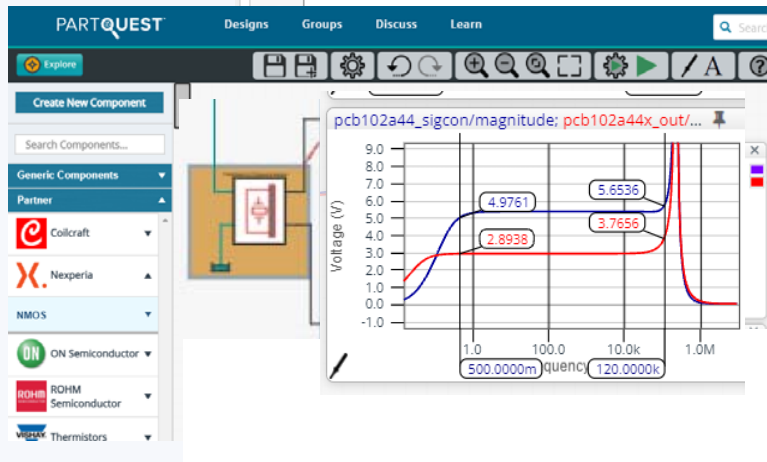
VHDL-AMS on Siemens PartQuest™ Explore

Code equations directly in VHDL-AMS

```

92
93 X == eps*(s_E/Cp)*f_p + d33*V;
94 Q == d33*f_p + Cp*V;
95 tau*tscale*Q'dot + Cp*V == 0.0;
96
97 f_mass == mval*tscale*tscale*X'dot'dot;
98
99 f_v == k_v*tscale*X'dot;
100 flow == 0.0;
    
```

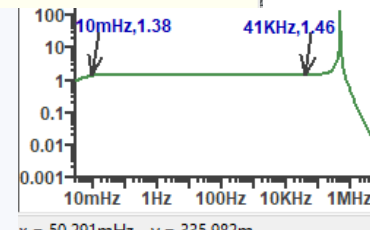
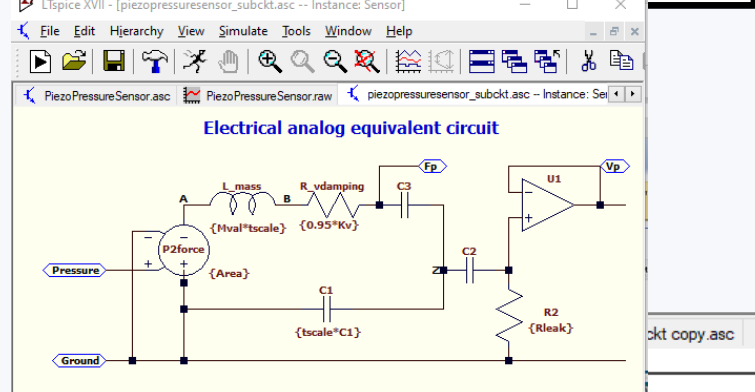
View or edit VHDL-AMS/



Component marketing,
product selection,
Collaborative Design

Equivalent Circuit on LTSpice
Use voltage, charge and current to represent mechanical force, Displacement and velocity

| | Mechanical | Electrical |
|---------|---------------------------------|---------------------|
| Across | Force | Voltage |
| Through | Velocity | Current |
| Mass | $F = m d(\text{velocity}) / dt$ | $V = m di/dt$ |
| Damping | $F = k_v \text{velocity}$ | $V = k_v i$ |
| Spring | Displacement $s = F/k$ | $i = (1/k_v) dv/dt$ |



Free download
Runs on your desktop

From Equations

$$P A + (m s^2 + k_v s + k_s) X + f_p = 0$$

$$X = s^2 X_0 \frac{A}{\omega} + d_{33} V$$

$$Q = d_{33} f_p + C_p V$$

$$T = \frac{1\text{-linearity}}{\omega_{\min} \sqrt{\text{linearity}(2\text{-linearity})}}$$

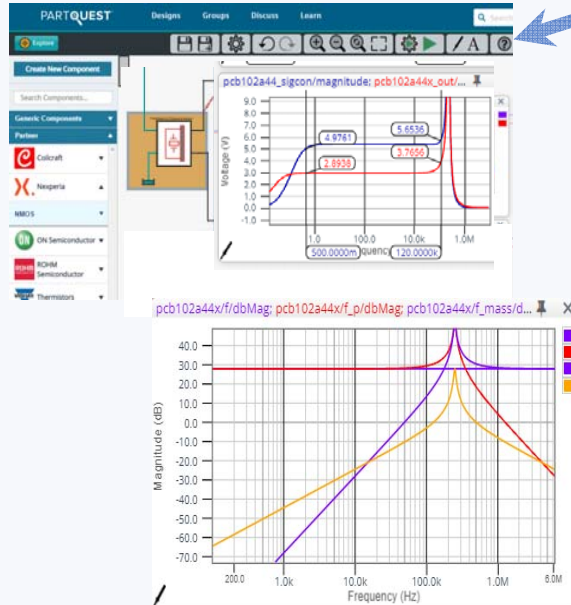
$$C_p = A d_{33} / \text{sensitivity}$$

$$m = \frac{C_p}{\omega_0^2 (\epsilon s^E - d_{33}^2)}$$

$$k_v = \frac{2 \zeta C_p}{\omega_0 (\epsilon s^E - d_{33}^2)}$$

to Simulations

VHDL-AMS on Siemens PartQuest™ Explore

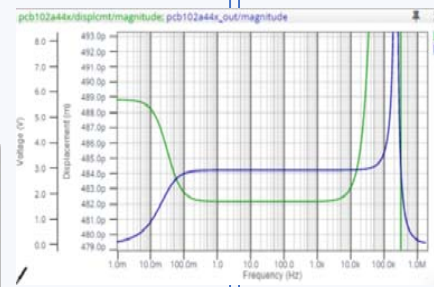
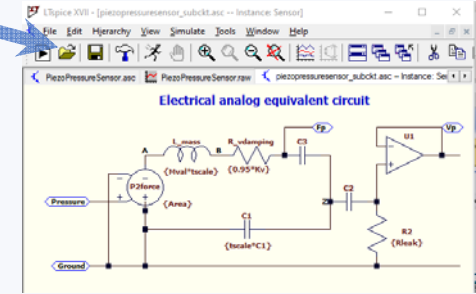


Displacement magnified to show low frequency pole-zero pair.

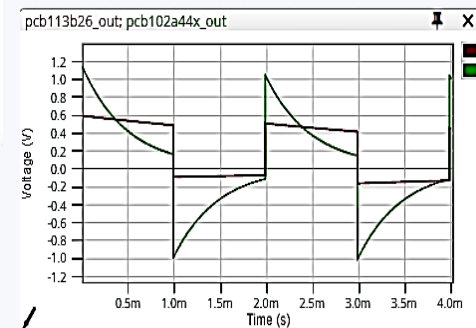
- Register free at <https://explore.partquest.com/>
- Open a copy of my test bench at <https://explore.partquest.com/groups/norms-workspace/designs/piezo-demo>
- Follow instructions to run simulations, save your own copy of the model.

via

Equivalent Circuit on LTspice



Comparison of forces: F_p dominates below $F_{r_{es}}$



Time domain responses to pulse train (20ns at 50% duty cycle) scaled to 1ms:10s.

- Download and install free copy of LTspice from <https://www.filehorse.com/download-ltspice/>
- Email me at normelias@outlook.com to request copies of the LTspice schematic.

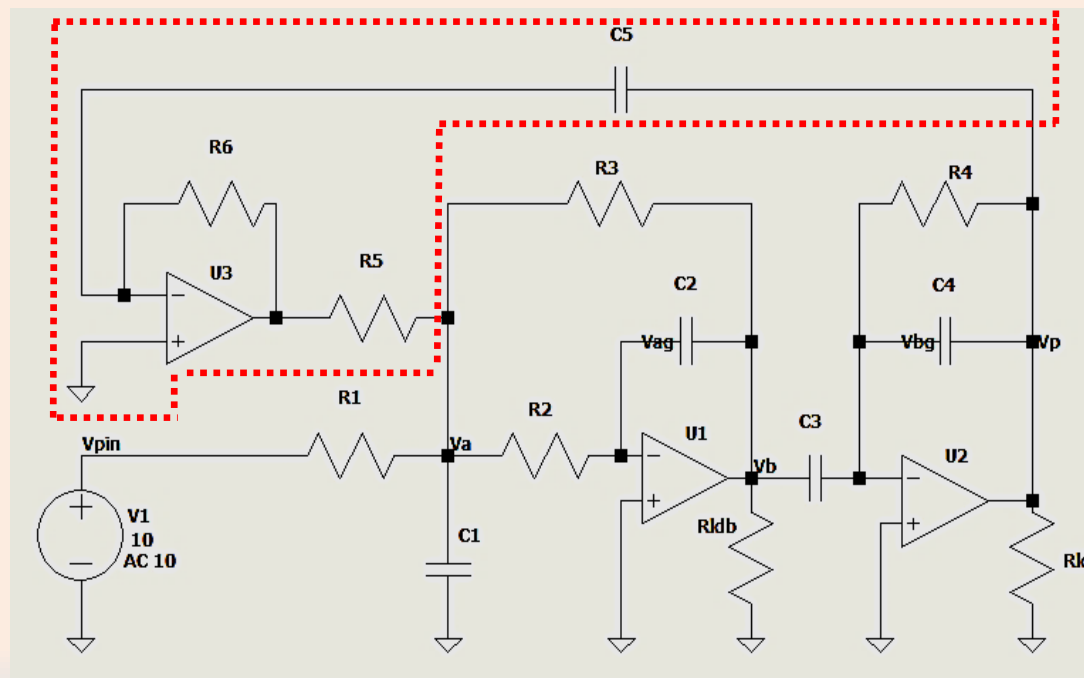
$$(\epsilon s^E m s^2 + \epsilon s^E k_V s + C_p) \left(1 + s\tau \left(1 - \frac{d_{33}^2}{\epsilon s^E} \right) \right) V + C_p \left(\frac{d_{33}^2}{\epsilon s^E} \right) s\tau V = s P A d_{33} \tau$$

$$(\epsilon s^E m s^2 + \epsilon s^E k_V s + C_p) \left(1 + s\tau \left(1 - \frac{d_{33}^2}{\epsilon s^E} \right) \right) V = s P A d_{33} \tau - C_p \left(\frac{d_{33}^2}{\epsilon s^E} \right) s\tau V$$

$$V = \frac{s P A d_{33} \tau - C_p \left(\frac{d_{33}^2}{\epsilon s^E} \right) s\tau V}{(\epsilon s^E m s^2 + \epsilon s^E k_V s + C_p) \left(1 + s\tau \left(1 - \frac{d_{33}^2}{\epsilon s^E} \right) \right)}$$

The extra term in red represents...

...negative **FEEDBACK!!**

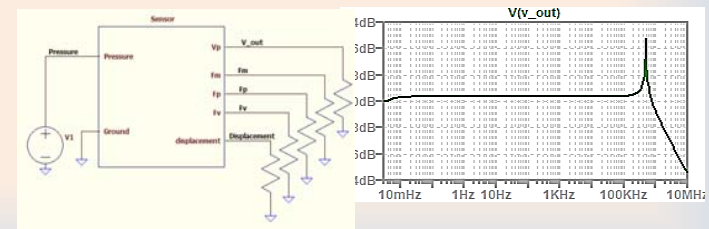
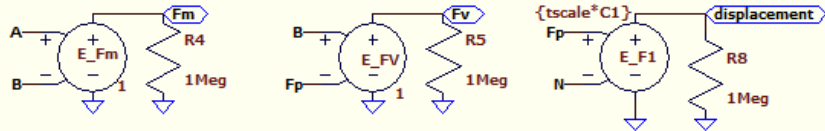
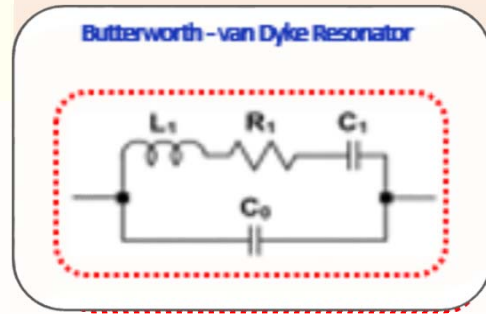
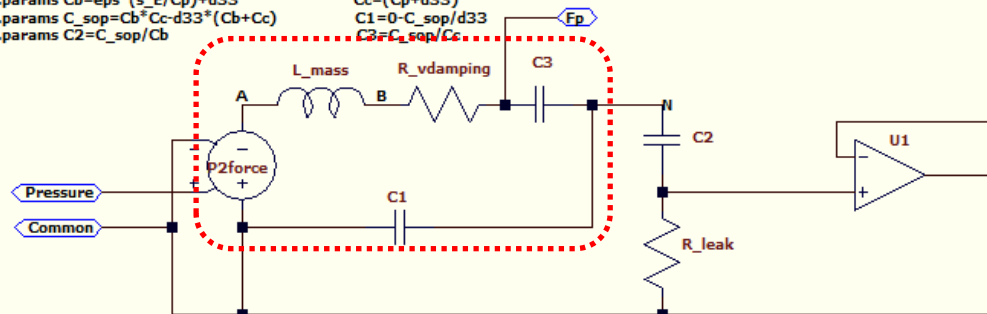


LTspice Equivalent Circuit Viewed as A Generalized Butterworth - van Dyke Resonator

```
.params epsrel=4.52          s_E=9.7e-12
.params d33=2.3e-12         diameter=0.218
.params eps=epsrel*1e-9/(36*pi) Area=pi*(0.5*diameter*0.0254)**2
.params f_min=0.01         f_res=500K          sensitivity=1.45e-6
.params w_min=2*pi*f_min   w_0=2*pi*f_res
.params linearity=0.05     zeta=0.03          tscale=1.0
.params kmin=sqrt(((1-linearity)/linearity)**((1-linearity)/(2-linearity)))
.params kappa_sq=(d33/eps)**(d33/s_E)    tau=kmin/w_min
.params Cp=Area*d33/sensitivity          Rleak=tau/Cp
.params mval=1/(((eps*w_0**2)**(s_E/Cp)-(d33*w_0**2)**(d33/Cp)))
.params Kv=2*zeta*w_0**mval
.params E_V2F=0-(Cp/eps)**(d33/s_E)     E_F2V=0-d33/Cp
.params Cb=eps*(s_E/Cp)+d33              Cc=(Cp+d33)
.params C_sop=Cb*Cc-d33*(Cb+Cc)          C1=0-C_sop/d33
.params C2=C_sop/Cb                     C3=C_sop/Cc
```

```
.ac dec 100 3m 10Meg
.LIB opamp.sub
```

Electrical analog equivalent circuit



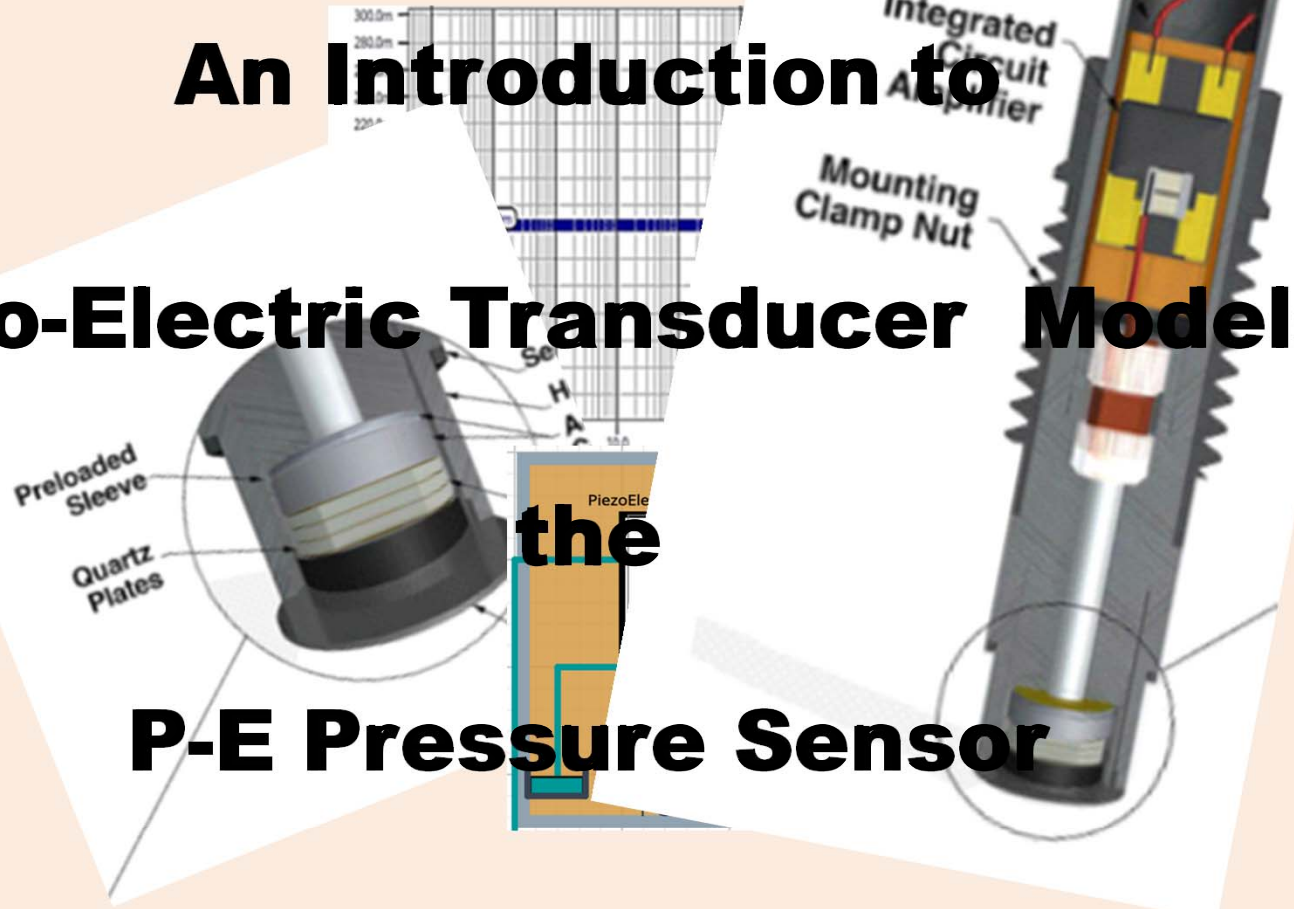
Summary

- **Express model equations in terms of datasheet specs**
 - Start with Mechanical structure + IEEE std
 - Solve for V/Pressure
 - Match Low Freq, mid-band, High Freq to datasheet
- **Three ways to implement the equations**
 - Direct in VHDL-AMS (PartQuest Explore online)
 - Analog equivalent circuit (Download LTspice)
 - Active filter stages (Potential Hardware Emulator)
- **Models Voltage, Current, Displacement, Forces**
- **Generalizes Butterworth-van Dyke Resonator Model**

Thanks for listening



An Introduction to Piezo-Electric Transducer Models: the P-E Pressure Sensor



Norman Elias

Addendum

The remainder of this powerpoint presents details of the model derivation and outlines key observations that do not fit well within the body of the slide show.

Addendum

Model Derivation:

On the mechanical side, the structure includes the piezoelectric element, a diaphragm to deliver the pressure uniformly to this piezoelectric element, and an inertial mass to stabilize the structure. There is also a spring structure to absorb the applied force, however the derivation is greatly simplified by subsuming that spring into the mechanical resistance of the piezoelectric element. Under that assumption, I'll introduce the following Laplace-transformed equation for the mechanical front end

$$P A + (ms^2 + k_v s)X + f_p = 0 \quad [2]$$

where X is the displacement caused by the applied pressure and f_p is the resistive force of the piezoelectric element.

The remainder of the model is a two-port linking the mechanical side to the electrical output of the pressure sensor. IEEE standard 176-1987 (See Standards Committee of the IEEE Ultrasonics, Ferroelectrics, and Frequency Control Society, 1987) provides a linearized model of the two-port model for small signal. In terms of Figures 3 and Figure 4, the two-port equations are

$$X = s^E X_0 \frac{f_p}{A} + d_{33} V \quad (\text{Converse Piezoelectric Effect}) \quad [3A]$$

$$Q = d_{33} f_p + C_p V \quad (\text{Direct Piezoelectric Effect}) \quad [3B]$$

where V is the voltage difference between the metal plates at either side of the piezoelectric element, Q is the net charge on those plates, and $C_p = \epsilon \frac{A}{x_n}$ is the formula for a parallel plate capacitance (neglecting fringing effects). The other terms in these equations are physical characteristics of the piezoelectric crystal, viz.,

s^E : Compliance in meter²/newton = 9.7e-12 for quartz

d_{33} : piezoelectric charge constant in coulombs/newton = 2.3e-12 for quartz

ϵ : Permittivity (in coulombs/meter²) = 4.5 * ϵ_0 = 1E-9/8 π , where ϵ_0 is the permittivity of free space

Model Derivation: (Continued)

We can assign values to the coefficients in Equations [2] and [3] by deriving the transfer function and comparing it to Equation [1]. This is done by using Equations [3] to express X and f_p as functions of V . Taking the derivative on both sides of [3B] and invoking Laplace transforms.

$$sQ = d_{33}sF_p + C_p sV = -\frac{sV}{R_{LK}}$$

where R_{LK} is the leakage resistance that terminates the electrical side. Solving for sF_p .

$$sF_p = -\frac{C_p}{d_{33}\tau}(1+s\tau)V$$

[4]

If we differentiate [3A] we can substitute [4] to get

$$sX = -s^E \frac{X_0}{A} \left(\frac{C_p}{d_{33}\tau}(1+s\tau)V \right) + d_{33}V$$

and by recognizing $C_p = \epsilon \frac{A}{X_0}$ we get

$$sX = -\frac{\epsilon s^E}{d_{33}\tau}(1+s\tau_2)V$$

[5]

$$\text{where } \tau_2 \triangleq \tau \left(1 - \frac{d_{33}^2}{\epsilon s^E} \right)$$

Next, we substitute [4] and [5] into Equation [2] and rearrange to get the transfer function.

$$\frac{V}{P} = \frac{A d_{33} \tau}{\epsilon s^E (ms^2 + k_v s) (1 + s\tau_2) + C_p (1 + s\tau)}$$

Model Derivation: (Continued)

Table 1 compares Equations [6] and [1] over two frequency ranges to assign numerical values to the physical parameters in [6]

| Frequency Range | From Equation [1] | From Equation [6] | Parameter Assignments |
|-----------------|--|---|---|
| Low to Mid-Band | $\frac{s(k_{min}/\omega_{min}) \text{ sensitivity}}{(1 + s(k_{min}/\omega_{min}))}$ | $\frac{s\tau(A d_{33}/C_p)}{(1 + s\tau)}$ | $\tau = k_{min}/\omega_{min}$ [7A] $C_p = A d_{33}/\text{sensitivity}$ [7B] |
| High | $\frac{(\omega_{min}/k_{min}) \text{ sensitivity}}{((s/\omega_0)^2 + 2\zeta(s/\omega_0) + 1)}$ | $\frac{A d_{33}/C_p \tau}{((\epsilon s^E - d_{33}^2)/C_p) m s^2 + (\epsilon s^E - d_{33}^2)/C_p k_v s + 1} (1 + s\tau)$ | $m = \frac{C_p}{\omega_0^2(\epsilon s^E - d_{33}^2)}$ [7C] $k_v = \frac{2\zeta C_p}{\omega_0(\epsilon s^E - d_{33}^2)}$ [7D] |

Table 1 Datasheet Parameter Assignments

To summarize, the equations that define the piezoelectric pressure sensor are as follows (expressed in the time domain) for an applied pressure P, a displacement X, an output voltage V, and a leakage resistance terminating the electrical output.

$$P A + m \frac{d^2 X}{dt^2} + k_v \frac{dx}{dt} + f_p = 0 \quad [8A]$$

$$X = \frac{\epsilon s^E}{C_p} f_p + d_{33} V \quad [8B]$$

$$Q = d_{33} f_p + C_p V, \quad \frac{dQ}{dt} = -\frac{C_p}{\tau} V \quad [8C, D]$$

where the parameters are defined by Equations [7].

Equivalent Circuits and SPICE-Compatible Structural Models:

In the absence of a hardware description language, the pressure sensor must be modeled as an equivalent circuit. The most direct approach is to base the model on the structure of the device¹. For SPICE and SPICE-compatible circuit simulators, this means realizing Equations [8A-D] as R-L-C circuits including electrical analogs for the mechanical portions of the model². For this purpose, I've chosen voltage (the "across" quantity) as the analog of force and current (the "through" quantity) as the analog of velocity so that Table 2 governs the mechanical portions of the model.

| | Mechanical | Electrical |
|-----------------|------------------------------------|------------------------|
| Across | Force | Voltage |
| Through | Velocity | Current |
| Mass | $F = m \, d(\text{velocity}) / dt$ | $V = m \, di/dt$ |
| Viscous Damping | $F = k_v \cdot \text{velocity}$ | $V = k_v \cdot i$ |
| Spring | displacement $= F/k_s$ | $i = (1/k_s) \, dv/dt$ |

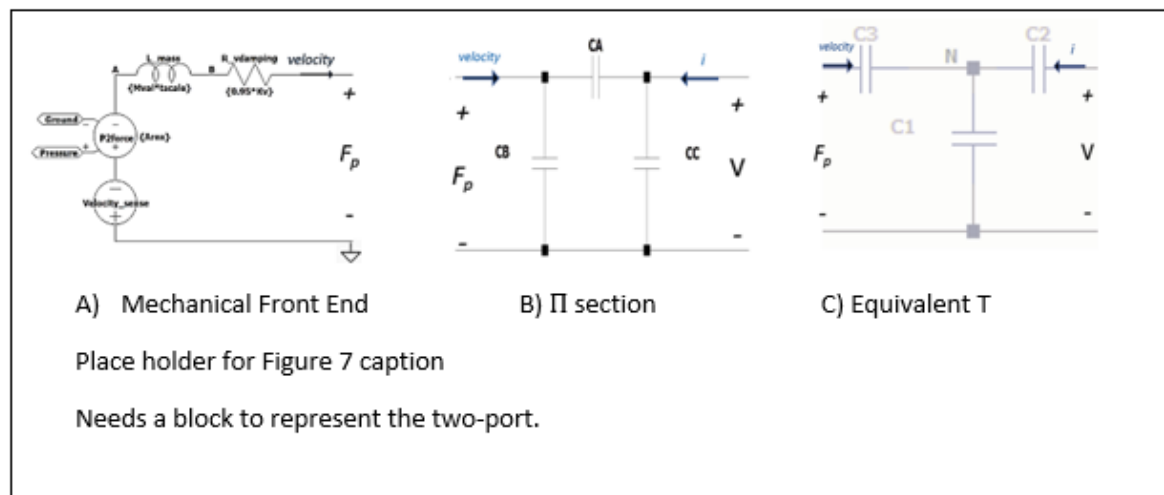
Table 2 Electrical Analog for the Mechanical Portion of the Model

Equation [8A] defines the mechanical front end in terms of the piezoelectric sensor's *displacement*. Using Laplace transform notation, Equation [9] expresses the same relation in terms of *velocity*.

$$\text{Pressure} * \text{Area} + s \, m \, \text{velocity} + k_v \, \text{velocity} + f_p = 0 \quad [9]$$

For the conventions in Table 2, Equation [9] represents the series R-L circuit shown in Figure 7A.

Equations [8B-D] define the two-port in terms of displacement and stored charge. Equation [10a and b] expresses these relations in terms of *velocity* and current.



$$velocity = s \left(\frac{C_p}{\epsilon s E} \right) F_p + s d_{33} V \quad [10A]$$

$$\frac{dQ}{dt} = i = s d_{33} f_p + s C_p V \quad [10B]$$

These equations define the capacitive Π section shown in Figure 7B. The capacitances are as follows³

$$\begin{aligned} CA &= -d_{33} \\ CB &= \left(\frac{C_p}{\epsilon s E} \right) + d_{33} \\ CC &= C_p + d_{33} \end{aligned} \quad [11]$$

Figure 7C shows an equivalent capacitive T section from which the displacement can be extracted from the voltage across C1. As discussed in the Observations to follow, the T section quantifies the decision to neglect physical springs in the mechanical structure. Clearly, that assumption is valid provided the spring constant (stiffness) is significantly less than $1/C_3$. The T-section capacitances are as follows.

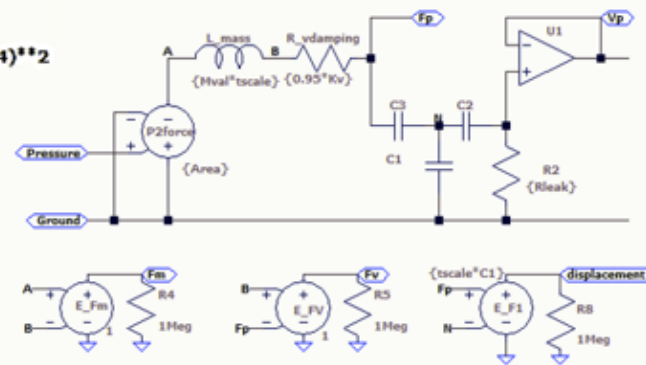
$$\begin{aligned} C1 &= \frac{CA CB + CB CC + CC CA}{CA} \\ C2 &= \frac{CA CB + CB CC + CC CA}{CB} \\ C3 &= \frac{CA CB + CB CC + CC CA}{CC} \end{aligned} \quad [12]$$

Figure 8 shows the completed model as implemented in LTspice. It includes the amplifier typically added to buffer the output voltage. The text added to the schematic defines the model parameters and incorporates Equations [7a-d] as well as the above capacitance expressions into the model. Appendix B contains a spreadsheet calculator that computes model components from the data sheet parameters.

```
.ac dec 100 3m 10Meg
.params epsrel=4.52
.params d33=2.3e-12
.params eps=epsrel*1e-9/(36*pi)
.params f_min=0.01 f_res=500K
.params w_min=2*pi*f_min
.params linearity=0.05 zeta=0.03
.params kmin=sqrt(((1-linearity)/linearity)^((1-linearity)/(2-linearity)))
.params kappa_sq=(d33/eps)^2*(d33/s_E)
.params Cp=Area*d33/sensitivity
.params mval=1/((eps*w_0**2)*(s_E/Cp)-(d33*w_0**2)*(d33/Cp))
.params Kv=2*zeta*w_0*mval
.params E_V2F=0-(Cp/eps)^2*(d33/s_E)
.params Cb=eps*(s_E/Cp)+d33
.params C_sop=Cb*Cc-d33*(Cb+Cc)
.params C2=C_sop/Cb
```

```
.LIB opamp.sub
s_E=9.7e-12
diameter=0.218
Area=pi*(0.5*diameter*0.0254)**2
sensitivity=1.45e-6
w_0=2*pi*f_res
tscale=1.0
P2Forced
(Area)
(Nval*tscale) (0.95*Kv)
tau=kmin/w_min
Rleak=tau/Cp
E_F2V=0-d33/Cp
Cc=(Cp+d33)
C1=0-C_sop/d33
C3=C_sop/Cc
```

Electrical analog equivalent circuit



³ Negative valued capacitances are acceptable except for C_p because none of the other capacitances represent physical capacitors. Referring to the mechanical analog, a negative capacitance models a spring under tension rather than compression.

Test Results and Confirmation of the LTspice Model:

To confirm the LTspice model, AC simulations were run for two significantly different sets of parameters, both selected from the PCB Piezotronics web site. Figures 9 and 10 confirm that the model successfully reproduces the frequency responses specified for these two pressure sensors.

MODEL: 113B26

Platinum Stock Products;High frequency ICP® pressure sensor, 500 psi, 10 mV/psi,
0.218" dia. diaphragm accel. comp.

- Measurement Range: (for ±5V output) 500 psi(3450 kPa)
- Sensitivity: (±10%) 10 mV/psi(1.45 mV/kPa)
- Low Frequency Response: (-5%) 0.01 Hz
- Resonant Frequency: >=500 kHz(>=500 kHz)
- Electrical Connector: 10-32 Coaxial Jack
- Weight: (with clamp nut) 0.20 oz(6.0 gm)

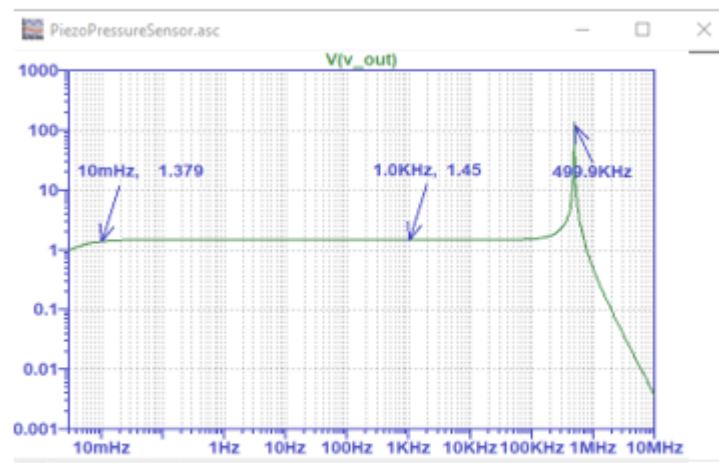


Figure 9 LTspice Simulation of PCB Piezotronics 113B26 Pressure Sensor Frequency Response

<http://www.pcb.com/nx/search-results?q=113b26>

<http://www.pcb.com/nx/search-results?q=105C02>

MODEL: 105C02

Subminiature ICP® pressure sensor, 100 psi, 50 mV/psi, 0.099" dia. diaphragm, 10-32 mtg thd

- Measurement Range: (for ±5V output) 100 psi(690 kPa)
- Sensitivity: (-40 to +20%) 50 mV/psi(7.3 mV/kPa)
- Low Frequency Response: (-5%) 0.5 Hz
- Resonant Frequency: >=250 kHz(>=250 kHz)
- Electrical Connector: 5-44 Coaxial
- Weight: 0.055 oz(1.56 gm)

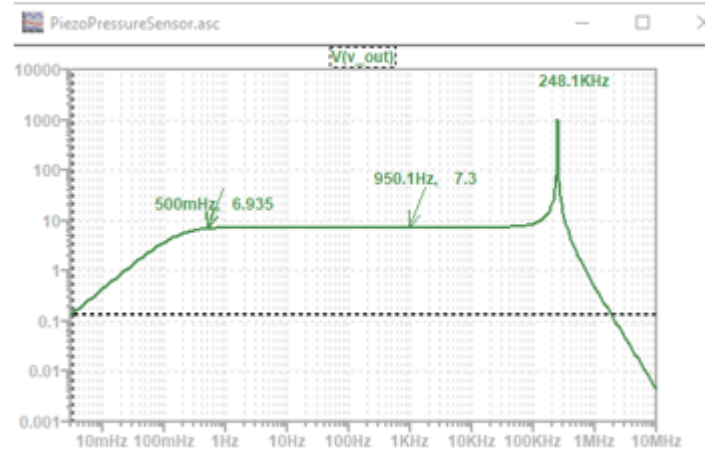


Figure 10 LTspice Simulation of PCB Piezotronics 105C02 Pressure Sensor Frequency Response